J. Phys. A: Math. Gen. **20** (1987) 6591-6592. Printed in the UK

COMMENT

Note on 'N-pseudoreductions' of the KP hierarchy

G Post

Faculty of Applied Mathematics, University of Twente, PO Box 217, 7500 AE Enschede, Netherlands

Received 27 July 1987

Abstract. The group-theoretical side of N-pseudoreductions is discussed. The resulting equations are shown to be easy transformations of the N-KdV hierarchy.

In Hirota (1986), so-called N-pseudoreductions of the KP hierarchy are introduced. Our purpose here is to give a group-theoretical interpretation, in the spirit of Segal and Wilson (1985). This leads to a description of the pseudoreductions in terms of the pseudodifferential operator

$$L = D + u_2 D^{-1} + u_3 D^{-2} + \dots \qquad D = \frac{\partial}{\partial x};\ u_i = u_i(x = t_1, t_2, \dots).$$

The KP hierarchy is given by

$$\partial L/\partial t_i = [(L^i)_+, L] \qquad (i \geq 1).$$

The τ-functions of the KP hierarchy are elements of the orbit of $1 \in \boldsymbol{C}[[t_1, t_2, \dots]]$ under the action ρ of a group G related to gl_∞. The elements of G are linear operators on a part of $\boldsymbol{C}[[t_1, t_2, \dots]]$. Examples of G can be found in Kac (1983) and Segal and Wilson (1985). G is a central extension of some $\tilde{G}$, $\tilde{G}$ being a group of $\boldsymbol{Z} \times \boldsymbol{Z}$ matrices.

Now for N-pseudoreduced τ functions, it holds (by definition)

$$\left(\frac{\partial}{\partial t_N} - c_0 \frac{\partial}{\partial t_1}\right) \tau(t_1, t_2, \dots) = 0. \tag{1}$$

Clearly $g \in \rho(G)$ with

$$\left[g, \frac{\partial}{\partial t_N} - c_0 \frac{\partial}{\partial t_1}\right] = 0$$

maintains this condition. Projecting G onto $\tilde{G}$ this condition reads

$$[A, \Lambda^N - c_0 \Lambda] = 0 \qquad \left(\Lambda^k = \sum_{i \in \boldsymbol{Z}} E_{i,i+k},\ A \in \tilde{G}\right). \tag{2}$$

The elements from G, for which the projection satisfies (2), will be our group G_{pr}, associated to the pseudoreduction (1).

Equation (2) has only trivial solutions (namely linear combinations of the Λ^i) if the group $\tilde{G}$ is contained in $\{A = (a_{ij}) | a_{ij} = 0 \text{ if } i - j > M, \text{ some } M\}$.

To find non-trivial solutions one has to pass to extensions as in Segal and Wilson (1985). Moreover, using this Grassmannian approach, one proves the following statements: an element of G determines an operator L; in the case $c_0 = 0$, condition (2) yields $L^N = (L^N)_+$, leading to the KdV-type hierarchies. In a similar way, one can prove that in the general case

$$L^N - c_0 L = (L^N - c_0 L)_+ =: P.$$

The following equation holds for P:

$$\frac{\partial P}{\partial t_i} = [(L^i)_+, P] \qquad (i \geqslant 1). \tag{3}$$

In particular, since $[(L^N)_+, P] = [c_0 D, P]$, we find that $u_{2,t_N} = u_{2,x}$. Calculating the $N = 2$ (modified) and $N = 3$ pseudoreductions, using the time flow of (3), we find exactly (4.7) and (4.3) of Hirota (1986).

By equation (3) it is clear that pseudoreductions will yield linear combinations of the canonical (i.e. graded) KdV-type equations. In this sense pseudoreductions of the KP-hierarchy are equivalent to the usual reductions, and yield nothing new.

References

Hirota R 1986 *Physica* D **18** 161
Kac V G 1983 *Infinite dimensional Lie Algebras* (Basle: Birkhauser)
Segal G and Wilson G 1985 *Publ. Math. IHES* no 61 p 5