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Correlation Effect in Probabilistic Design against Piping in Multi-Functional Flood Defences

J.P. AGUILAR LÓPEZ [a], J.J. WARMINK[a], R.M.J. SCHIELEN[a,b] and S.J.M.H. HULSCHER[a]
[a] *Twente Water Center - University of Twente, The Netherlands*
[b] *Ministry of Infrastructure and Environment (Rijkswaterstaat), The Netherlands*

Abstract. The multi-functional flood defences are one of the many solutions proposed for climate adaptation in deltaic areas. Similar concepts flood management structures such as Delta-dikes, un-breachable dikes or super-levees can also be found in literature. Yet all of them have two key design requirements in common. The first one is to improve their reliability given the uncertainty in future projections of water level change. The second one is the optimization of their dimensions in order to make them safe enough . In the case of the Netherlands, the failure mechanism of piping is one of the main concerns of the designers and flood risk managers. The large uncertainties that come from the heterogeneity of the soil properties are coped by moving towards a more complete probabilistic design. However correlation between input parameters is one of the sources of uncertainty that must be solved. In the present study, the effect of possible correlation between grain size and hydraulic conductivity is investigated. A straight forward approach is implemented by the inclusion of joint bivariate distributions generated from copulas. This random sampling method allows to induce not only the degree of correlation of two variables but their different tail dependence behavior as well. The generated random samples are propagated with a Monte Carlo algorithm using the piping Sellmeijer limit state function in order to estimate the reliability of the analyzed flood defence. The present study showed that correlation has an important effect in the design of this kind of structures which is even more severe when implemented according to the actual Dutch legislation. Therefore a robust soil investigation can be more cost effective than the actual over dimensioning of the structures.

Keywords. Piping, Correlation, Copulas, Flood defences, Sellmeijer

1. Introduction

In the Netherlands, projects like the VNK (Jongejan, et al. 2013) found that piping erosion was one of the most important failure mechanisms for the Dutch flood defence system. This type of failure consists in a progressive erosion under the flood defence which will eventually compromise the stability of the structure. Multifunctional flood defences (*MFFD's*) are one of the many solutions proposed for urban areas where factors such as sea level rise and global warming are making the existent flood defences less reliable against failure mechanisms such as piping. These structures should be designed to withstand loading events with higher return periods than the ones used for the design of the existent flood defences. Therefore, their required enlargement of their dimensions is exploited for additional functions and infrastructure embedment. Note that the minimum dimensions are defined by the required safety levels and not by the additional functions space requirement.

Extreme flood events for which flood defences are designed, have a tendency to occur as a combination of other events. Diermanse and Geerse (2012) for example, showed that the correlation inclusion in the estimation of the load term of models used for dike reliability such as levels and wind speed, might prevent the underestimation of failure probabilities by orders of magnitude. Likewise, correlations can also be present in processes that represent the resistance to such extreme loads.

In the present paper, the effect of the inclusion of correlation between the two sensitivity parameters involved in the Sellmeijer piping erosion limit state equation is explored. Its effect on the results on MFFD dimensioning (Cross section seepage length) is studied by implementing a Clayton copula bivariate model for different correlation degrees.

2. Piping Erosion

Piping erosion consists in the loss of stability of the flood defence structure due to the erosion of the granular foundation stratum. In order for piping to happen, a previous failure mechanism called "uplift" must have occurred. This failure mechanisms consists in the increase pressures during a flood event to a level where they exceed the weight of the upper impermeable layer that eventually will trigger its cracking. If so, an exit point is originated in which the aquifer is exposed to the atmospheric pressure which allows the water to flow freely outside of its confinement. Hence for piping to occur, uplift needs to occur as well.

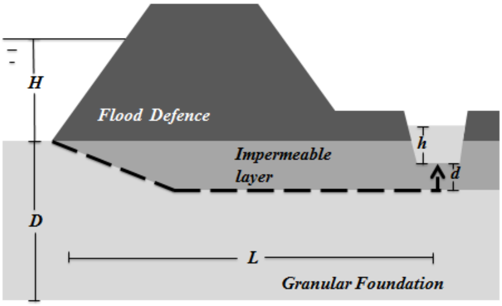

Figure 1. Uplift/Piping erosion

Afterwards, the water flow inside the aquifer towards the inland side, will transport sand grains originating a longitudinal cavity also referred as "pipe". In terms of limit state, if such pipe has developed for a length equal or greater than half the width (L) of the flood defence (Figure 1), it is assumed that the stability of the structure in terms of piping is in a failure state.

Several empirical models for estimating piping erosion state have been developed since the early 20th century. In the Netherlands, the numerical model by Sellmeijer (1991) for calculating the critical head was transformed and fine-tuned (Sellmeijer, et al. 2011) in the form of a limit state equation (Eq. (1) to Eq.(5)). This equation compares the estimated hydraulic head against the maximum allowable head for the sand grains to be dragged. Note that, there is an uncertainty model factor (mp) which accounts for all the additional variations lost in the simplification of the process.

$$Z_p = H_c - (\mathrm{h} - \mathrm{h_b} - 0.3\mathrm{d}) \tag{1}$$

$$H_c = m_p(F_G)\,(F_R)\,(F_S)\mathrm{L} \tag{2}$$

$$F_S = \frac{\mathrm{d_{70m}}}{\sqrt[3]{\left(\frac{\nu K}{g}\right)\mathrm{L}}}\left(\frac{\mathrm{d_{70}}}{\mathrm{d_{70m}}}\right)^{0.4} \tag{3}$$

$$F_R = \eta\,\frac{\gamma'_{\mathrm{sand}}}{\gamma_{\mathrm{w}}}\,\tan(\theta) \tag{4}$$

$$F_G = 0.91\left(\frac{\mathrm{D}}{\mathrm{L}}\right)^{\left(\frac{0.28}{\left(\frac{\mathrm{D}}{\mathrm{L}}\right)^{2.8}-1}+0.04\right)} \tag{5}$$

Table 1. Variables used in Sellmeijer equation

Var.	Unit	Description
Zp	[m]:	Limit state
η	[-]:	White's drag constant
γ'_{sand}	[N/m^3]:	Sand dry specific weight
γ_w	[N/m^3]:	Water specific weight
θ	[deg.]:	Sand rolling angle
d_{70}	[m]:	70% quantile grain size
d_{70m}	[m]:	Grain calibration size
ν	[m^2/s]:	Kin. water viscosity
K	[m/s]:	H. conductivity
g	[m/s^2]:	Gravity
D	[m]:	Foundation depth
m_p	[-]:	Uncertainty coefficient
Hc	[m.]:	Critical Head
FR	[-]:	Resistance factor
FS	[-]:	Scale factor
FG	[-]:	Geometric factor
L	[m]:	Defence width
h	[m]:	Water level
h_b	[m]:	Ditch water level
d	[m]:	Upper layer depth

For this study, the probability of uplifting was 3.92e-2. This value remains constant for all results in this study. The complete uplift limit state equation used in probabilistic piping assessment is explained in (Schweckendiek, et al. 2014).

3. Correlation Modeling

For the latest re-calibration of the Sellmeijer limit state equation, a multivariate analysis performed for small scale experiments (Sellmeijer, de la Cruz, van Beek and Knoeff 2011) showed that the hydraulic permeability K and grain size d70 parameters have the most strong influence in the critical head estimation variance. Therefore correlation between these two parameters will have an important effect in the output if modeled as bivariate jointly distributed.

3.1. Copula Joint Distributions

One of the different ways of implementing the correlation effect of two desired variables in a reliability study, is by using copula methods during the random sampling process. They allow to build joint distributions from two or more variables while maintaining the statistical properties of their original marginal distributions. For the case of bivariate distribution, the general copula probability function can be expressed as:

$$F_{X,Y}(x, y) = C[F_X(x), F_Y(y) \mid \theta] \qquad (6)$$

Where $F_X(x)$ and $F_Y(y)$ represent the cumulative distribution functions of the random variables x and y. The θ symbol represents the correlation copula parameter which describes the degree of dependence between variables x and y, and $F_{X,Y}$ is the Joint probability function given x and y.

A random sampling procedure from this functions consists in generating copula correlated values between 0 and 1 that follow a uniform distribution. These values are equivalent to the marginal probabilities of occurrence of each of the sampled random variables which are later transformed to the random numbers which follow the assumed distribution of each parameter. These generated values are the ones used for the Monte Carlo evaluations for the probabilistic assessment. Several different parametric copula family types can be found in the literature (Nelsen 1999), depending on a desired tail dependence behavior. The most common types of copula families are the “Gaussian” and the “Archimedean” functions (e.g Gumbel, Frank or Clayton). Their choice, allow to emphasize the tail dependence topologically speaking.

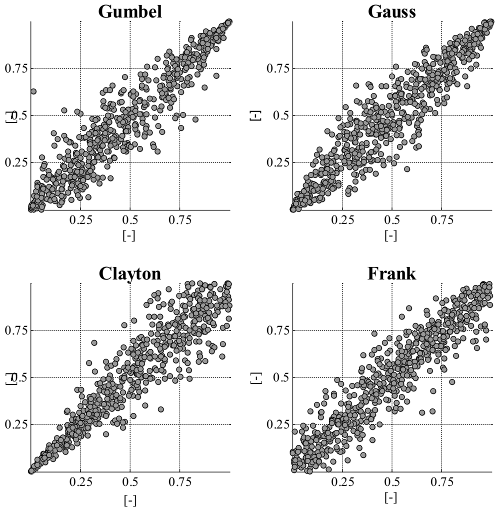

Figure 2. Copula Family for bivariate case

3.2. D70 and K Correlation

Extensive research has been done concerning the estimation of hydraulic conductivity based on the grain size distribution of the soil (Chapuis 2012). This allows us to suspect that the hydraulic conductivity (K) and representative grain size (d70) used in the Sellmeijer model could be correlated given the fact they describe the same aquifer. If so, two major characteristics were extracted as mandatory for the K vs d70 bivariate model to be representative. First, the model is positively correlated given the large evidence found in the literature. Second, the correlation should be stronger for the smaller values as most of the models describe the hydraulic conductivity based on the d10 quintile and even experimental data shows better agreement for low sized grained soils (Arya, et al. 1999). Based on these two characteristics, the present study assumes that the *Clayton Copula* (Figure 2) correlation model is capable of representing better the relationship between de d70 grain size diameter and the hydraulic conductivity parameter used in the Sellmeijer limit state equation.

During the piping erosion process, only the most upper part of the aquifer is eroded which means that the d70 statistical distribution should be representative of mainly that zone. Additionally, it is common to find finer grains in the upper layer of the aquifers which will imply the nonexistence of correlation between de d70 and the K parameters. However, in the actual practice such a sampling procedure is burdensome. Then, the d70 and K statistical descriptors are estimated from all the available sand samples extracted indistinct from their depth as long as they belong to the aquifer immediately underneath the impermeable layer.

4. Probabilistic MFFD Design

For the piping failure mechanism case, the most efficient countermeasure for increasing the flood defence resistance, is to extend the structure bottom width (Feasible seepage length). This is one of the main technical reasons of implementing robust flood defence concepts such as the Delta dike, the Unbreachable dike, the Robust dike or the MFFD's (Silva and van Velzen 2008, van Loon-Steensma and Vellinga 2014). In the case of the MFFD's, the enlargement of the cross section (Design dimension) is constrained by the failure probability and not by the additional space required for not primary defence functions.

A Clayton Copula is assumed for the correlation modeling as it allows to express the degree of dependence of the two variables in terms of the Kendall's rank correlation coefficient (τ). Such coefficient measures the degree of dependence based on how many data points are concordant compared with the ones that are not concordant. This condition ensures that the dependence degree will not be affected by any transformation in the original data set. The degree of correlation is going to be changed in steps of 0.1 staring from 0 until 1.0. This will allow to obtain a different designs (width of the cross section) for each flood defence width depending on different correlation degrees. The present design method estimates the required width of the MFFD based on the selected degree of correlation between d70 and K parameters present in the Sellmeijer equation scale factor F_s (Eq. 3). An hypothetic MFFD is going to be designed for a flood defence cross section of the ring 16 located below the Utrecht Province. The statistical distribution of the soil parameters (Table 2) are assumed as the ones used in the VNK (Vergouwe, et al. 2014) for dike ring 16 in a location where a strengthening measure was recommended.

Table 2. Soil properties statistical parameters

Var.	**Unit**	**Dist. Type**	**Mean**	**C.V.**
η	[-]	Constant	0.25	-
γ'_{sand}	[N/m^3]	Normal	16	1%
γ_w	[N/m^3]	Constant	9.81	-
θ	[deg.]	Constant	37	-
d_{70}	[m]	Log-normal	3.33e-4	15%
d_{70m}	[m]	Constant	2.08e-4	-
K	[m/s]	Log-normal	3.00e-4	100%
D	[m]	Log-normal	65	10%
mp	[-]	Log-normal	1	12%
L*	[m]	Log-normal	70+Δ	10%
H**	[m]	Gumbel	a=4.357	b=0.288
h_b	[m]	Normal	0.5	10%
d	[m]	Log-normal	7.5	30%

*Δ is equivalent to steps of 10 meter variation in until a final width of 160 meters.

**The load term H is assumed to follow a Gumbel extreme distribution with shape and location parameters equal to a and b.

There is no legislation or policy developed yet that states which target reliability index is required for the implementation of MFFD's. Therefore for the present study, the unbreachable dike safety level proposed by Silva and van Velzen (2008) is taken as the required target reliability for MFFD's. Their concept states that the safety level should be 100 times smaller than the required annual target failure probability of a traditional Dutch flood defence. Note that this required value is defined for the total aggregated failure probability of the whole defence. This means that it includes the combined effect of uplift/piping with other important failure mechanisms that can be triggered during a flood event and in different representative cross sections. For the present study it is also assumed that the other possible failure mechanisms (e.g. overtopping or macro-stability) are independent among each other and that the whole defence has the same representative cross section. This means that the flood defence can be treated as a series system where the total probability of failure is equivalent to the summation of the probability of each of the considered failure modes for one cross section.

Results from the VNK project safety assessment for the Dike ring 16, showed that 79.4% of the total estimated failure probability could be attributed solely to the uplift/piping failure mechanisms in this particular ring. Therefore, the minimum annual estimated reliability index for uplift/piping would be:

- **VNK-MFFD**: (5e-4*0.794)/100 = 3.97E-6 or β= 4.467

In reality for the design of an actual flood defence and according the new Dutch flood risk legislation, the percentage maximum allowable contribution from the uplift/piping mechanism to the system's overall probability of failure is 24%.

- **Target-MFFD**:(5e-4*0.24)/100=1.2E-6 or β= 4.72

5. Results

In the different designs it is observed that or a constant length, the variation in the rank correlation (τ) does not affect the mean value of the limit state marginal distribution (Zp).

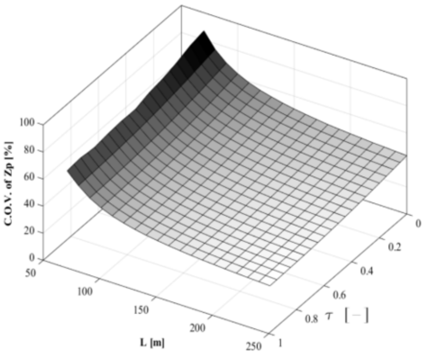

Figure 3. Coefficient of variation as a function of correlation and flood defence length

On the other hand, there is a significant change in the standard deviation. This is shown in figure 3, where the coefficient of variation of the limit state marginal function (Zp) can change by almost 40% in average between the uncorrelated (τ=0) and the totally correlated case (τ=0.99) no matter the chosen width. The impact in the standard deviation will change the failure probabilities in a nonlinear way as its effect is greater in the events located in the tail of the limit state marginal distributions. For the Sellmeijer model in particular, the MFFD safety increases proportionally and monotonically to the increase in correlation for each of the design widths (figure 4).

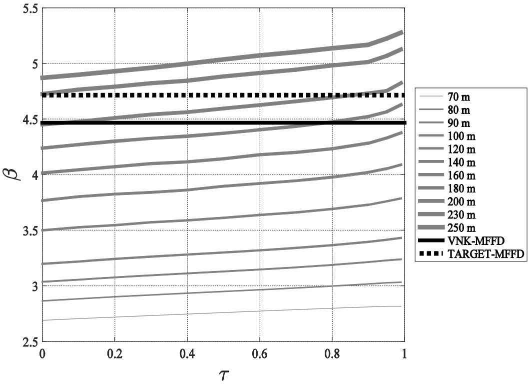

Figure 4. β indexes of different designs for ranging correlation degrees (τ)

6. Discussion

To analyze the obtained results, the correlation value between d70 and K is required. However, it can only be accurately estimated from soil sampling and in situ conductivity measurements. Nevertheless, a 22 borehole sample dataset of a similar aquifer provided in the research paper from Vienken and Dietrich (2011) was used to estimate the order of magnitude of such correlation. The obtained Kendall's rank correlation degree between d60 and their field measurements of hydraulic conductivity was τ = 0.443 (ρ=0.619). These samples were obtained from in situ field slug tests measurement on a highly heterogeneous unconsolidated aquifer in Bitterfield, Germany. Note that d60 and d70 are normally highly correlated for this type of granular soils.

In Figure 4, the continuous horizontal line corresponds to the actual required beta index and the continuous line represents the required beta index from the results obtained from VNK. If a similar correlation degree as the one estimated for the Bitterfield aquifer was observed for the hypothetic MFFD, a 190 meters width would be required in order to cope with the target reliability index estimated in the VNK project ($\beta = 4.467$). When assumed uncorrelated, a 200 width MMFD will be required instead for the same target reliability. Note that the increase in the defence width on each design, also affects the slope of the lines that represent their different reliabilities for each correlation value (Figure 4). This effect makes the correlation inclusion even more important for the design defences with higher target reliability indexes (wider cross sections). For new flood defences where there is no prior knowledge of the contribution of uplift/piping erosion to the total failure probability, the required widths are even greater. For the uncorrelated case, the flood defence will require a minimum width of 230 meters. This design is 15 meters wider than the one required when correlation is present ($\tau = 0.443$). Such a difference in a longitudinal structure as a flood defence, can easily determine its financial feasibility of the project since the start.

7. Conclusion

Correlation between d70 and K has a great impact in the design of MFFDs and conventional flood defences if proven. In fact, this effect is even more important for MFFD design as they are conceived as wider flood defences which means they have higher reliability indexes where the effect is more severe. Hence, the assumption of d70 and K to be non correlated plus implementing the actual safety standards can produce massive non-cost effective structures for the case of MFFD's. Therefore, robust soil investigation is recommended for large scale projects such as MFFDS as it may represent a substantial cost reduction. This solution will not imply great additional investigation costs for MFFD's where dense soil investigation will be available given the inclusion of the additional functions.

8. Acknowledgements

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