

# Control of slippage with tunable bubble mattresses

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**Tailoring the hydrodynamic boundary condition is essential for both applied and fundamental aspects of drag reduction. Hydrodynamic friction on superhydrophobic substrates providing gas-liquid interfaces can potentially be optimized by controlling the interface geometry. Therefore, establishing stable and optimal interfaces is crucial but rather challenging. Here we present unique superhydrophobic microfluidic devices that allow the presence of stable and controllable microbubbles at the boundary of microchannels. We experimentally and numerically examine the effect of microbubble geometry on the slippage at high resolution. The effective slip length is obtained for a wide range of protrusion angles,  $\theta$ , of the microbubbles into the flow, using a microparticle image velocimetry technique. Our numerical results reveal a maximum effective slip length, corresponding to a 23% drag reduction at an optimal  $\theta \approx 10^\circ$ . In agreement with the simulation results, our measurements correspond to up to 21% drag reduction when  $\theta$  is in the range of  $-2^\circ$  to  $12^\circ$ . The experimental and numerical results reveal a decrease in slip length with increasing protrusion angles when  $\theta \geq 10^\circ$ . Such microfluidic devices with tunable slippage are essential for the amplified interfacial transport of fluids and particles.**

Despite more than two decades of intense research on hydrodynamic slippage on substrates with various physicochemical properties (1–4), tuning the hydrodynamic slippage remains a challenge, especially for microfluidic laminar flow. The slip length—quantifying the slippage—ranges from a few nanometers for flat hydrophobic substrates to several micrometers for superhydrophobic substrates with hybrid (liquid–gas and liquid–solid) interfaces (4). Hydrophobic microstructures containing trapped gas bubbles have been shown to be advantageous for drag reduction (5–11). Their orientation with respect to the flow direction (12–16) and the geometry of gas–liquid menisci (11–14, 17) has been demonstrated to affect the slippage. In particular, microscale bubbles transverse to the flow direction can alter the flow resistance, depending on the protrusion of the bubbles into the flow. Moreover, transition from slippage to friction has been predicted for trapped bubbles perpendicular to the flow in theoretical (14, 18) and numerical studies (12, 13, 19, 20). The presence of such a critical protrusion angle highlights the feasibility of manipulating the flow resistance via bubble geometry. One recent experimental study suggests that for flow over a hydrophobic surface with trapped passive microbubbles, there is a transition from an enhanced slippage state to the frictional state at a large protrusion angle, in an estimated range of  $30^\circ$ – $60^\circ$  (20). However, there has been no experimental investigation of flow past a hydrophobic surface with transversely embedded microbubbles for a wide range of protrusion angles at high resolution. In this paper, we report on integrated microfluidic devices that permit the presence of stable and controllable microbubbles at the boundary of hydrophobic microchannels. We further examine in detail the effect of geometry of the microbubbles transverse to a pressure-driven flow on the effective slippage.

## Results and Discussion

We designed and fabricated microfluidic devices consisting of two main parallel microchannels for separate liquid and gas

streams, connected by an array of side channels in between (Fig. 1). To prevent the side channels from wetting, the original hydrophilic silicon microchannels are hydrophobized on the basis of silane chemistry, using perfluorinated octyltrichlorosilane (21). Due to the hydrophobicity of the surface and for sufficiently large applied gas pressure  $P_g$ , the bridging side channels are filled with gas. The control of the gas pressure results in tunable protrusion angles  $\theta$  of the microbubbles and will compensate for gas dissolution into the fluid (Fig. 1 *B* and *C*). Establishing stable and controllable bubbles is crucial in this study, which currently is attained by active control of gas pressure. In contrast, trapped passive bubbles are unstable as shown in previous studies (6, 11). The shape and the stability of the interface are determined by capillary forces and the pressure difference between the gas and liquid phases (8).

This fluidic configuration allows for easy and precise control of the gas–liquid interface curvature. It also enables visualization of both the interface geometry and the flow field near the bubble surfaces by direct velocity measurements of a steady, laminar flow past the microbubbles, using microparticle image velocimetry ( $\mu$ PIV) (22).

Numerically we study the effect of the interface shape on slippage, using 2D finite-element methods (Comsol Multiphysics). We solve for the pressure-driven flow of water for the flow settings used in the experiments with a computational domain representing the experimental geometric parameters (Fig. 1*D*). Shear-free boundary conditions are applied along the gas protrusions, whereas no-slip boundary conditions are imposed on the solid walls. Fig. 1*D* represents the computed flow field for  $\theta \approx 35^\circ$  over a bubble unit with length  $L$ , consisting of one no-slip (solid–liquid) and one shear-free (gas–liquid) boundary condition.

In Fig. 2, we present the flow velocity profiles measured in the middle of the microchannel depth by  $\mu$ PIV, providing direct quantification of effective slippage for varying geometric parameters: the protrusion angle,  $\theta$  and the shear-free fraction,  $\phi$ . In Fig. 2*A*, a representative bright-field image of a bubble unit with a protrusion angle of  $\theta \approx 43^\circ$  is presented. Fig. 2*B* represents the velocity field for  $\theta \approx 43^\circ$  protruding curved gas–liquid menisci, superimposed on a correspondent raw  $\mu$ PIV image. The bubble position and bubble curvature are depicted by dashed lines (Fig. 2 *A* and *B*). The curvature of the bubbles is calculated by circular arc estimation. The bubbles are stable and symmetric during the experiments due to sufficiently low capillary and Weber numbers [capillary ( $Ca$ )  $\approx 5 \times 10^{-3}$ , Weber ( $We$ )  $\approx 6 \times 10^{-3}$ ]. The bubbles' protrusion angles were accurately measured from the corresponding bright-field images with ImageJ analysis. The error bars were calculated from the standard deviation of  $\theta$  for each bubble. The locations and profiles of four successive bubbles, having

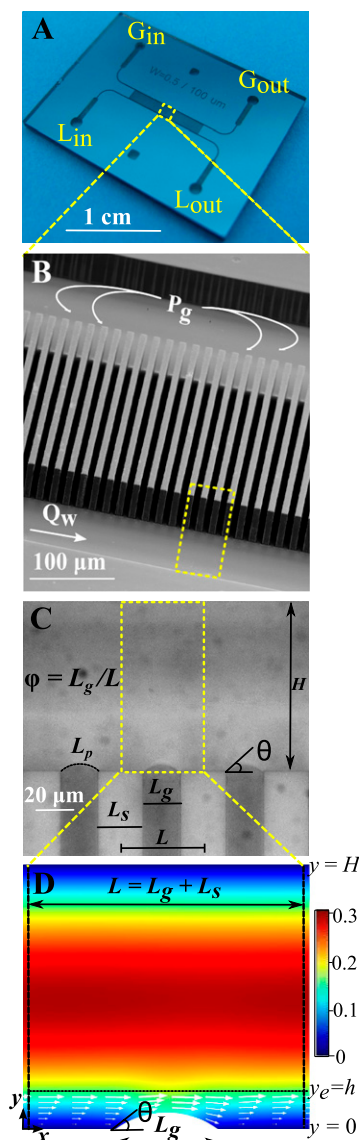
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**Fig. 1.** Controllable microfluidic bubble mattress and computational bubble unit cell. (A) Optical image of the microfluidic device with integrated gas (G) and liquid (L) channels, with the inlets and outlets indicated. (B) Scanning electron microscopy image of a representative microfluidic device, showing two main microchannels for gas ( $P_g$ ) and liquid ( $Q_w$ ) streams connected by gas-filled side channels. (C) Bright-field microscopy image of bubbles protruding  $35^\circ \pm 3.3^\circ$  into the liquid microchannel with a height,  $H$ . Here, the shear-free fraction,  $\phi$ , is defined as  $L_g/L = L_g/(L_s + L_g)$ , where  $L_g$  is the width of the gas gap, and  $L_s$  is the width of the solid boundary. (D) Numerical results of the pressure-driven flow over a microbubble unit, using the same experimental parameters. The color bar refers to the velocity, which is given in meters per second. Here  $\phi = 0.38$  and  $Q_w = 45 \mu\text{L}/\text{min}$ .

protrusion angles of  $43^\circ$ , are represented in Fig. 2C. The curved gas–liquid interfaces protrude  $\sim 3.7 \mu\text{m}$  into the microchannel in the  $y$  direction. A series of detailed velocity profiles,  $u_x(y)$ , at different channel heights,  $y$ , is presented in Fig. 2D. The presence of the microbubbles has a strong effect on the detailed velocity field. Near the wall, the variation of velocity field reflects the local variation of the hydrodynamic boundary condition. Even at  $y = 0.15H$  ( $y = 7.6 \mu\text{m}$ ), the effect of the bubbles on the flow field is still observed. Large protruding bubbles, as in this case ( $\theta \approx 43^\circ$ ), can act as obstacles to the flow. Indeed, a slight deceleration of the flow is observed in front of the bubbles. Right behind the

bubble, the loss in velocity is recovered and higher velocities are achieved. The overall average velocity in this case is still higher than that for nonslippery walls. These observations are consistent with the numerical findings of Hyvälöoma et al. (13).

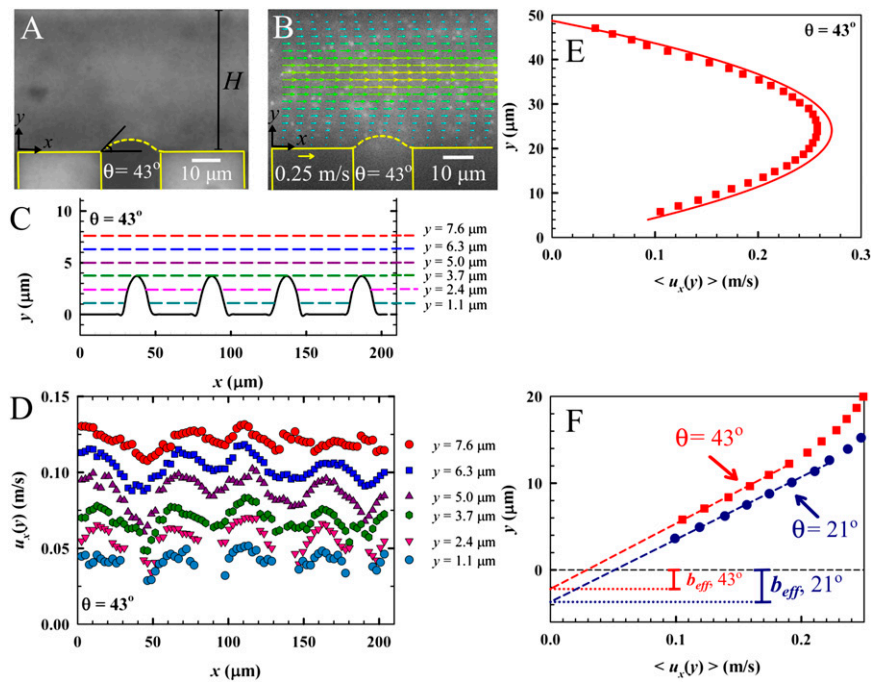
Fig. 2E presents the experimental and numerical velocity profiles obtained by the streamwise average of local velocity profiles. The measured and computed velocity profiles agree very well. The average velocity profiles near the hybrid wall are presented in Fig. 2F for the same experiment with  $\theta = 43^\circ \pm 6^\circ$  and for another experiment with  $\theta = 21^\circ \pm 3^\circ$ . Near the hydrophobic surface embedded with curved gas–liquid interfaces, the velocity profiles exhibit a linear relation between  $\langle u_x(y) \rangle$  and  $y$ . The effective slip length,  $b_{\text{eff}}$ , based on the average velocity profiles is calculated using Navier’s slip boundary condition (4):  $u_x|_{(y=0)} = b_{\text{eff}}(du_x/dy)|_{(y=0)}$ . To obtain  $du_x/dy$ , a least-squares linear fitting was performed for the data below  $y \approx 10 \mu\text{m}$ . The effective slip length calculation is graphically represented in Fig. 2F. The effective slip length for bubbles protruding  $43^\circ$  is smaller than that for those protruding  $21^\circ$ . Although slight deceleration of the flow is observed in front of  $43^\circ$  protruding bubbles (Fig. 2D), the effective slip length is found to be  $1.8 \pm 0.1 \mu\text{m}$  for this  $\theta$ , showing that  $43^\circ$  protruding bubbles still contribute to enhanced slippage. The numerical effective slip lengths were evaluated using the same approach with the local velocity gradients being calculated at  $y = 6.5 \mu\text{m}$ .

To further verify our methods, we compare our results with the analytical asymptotic solutions of the slip length for Stokes flow past periodically alternating, flat (protrusion angle  $\theta = 0^\circ$ ) shear-free and no-slip regions transverse to the flow (5) (with details provided in Appendix). Both the experimental and the numerical results of  $b_{\text{eff}}$  at  $\theta \approx 0$  are consistent with the predictions of the analytical asymptotes of the ratio of shear-free fraction.

In Fig. 3A, the first measurements of effective slip lengths for a wide range of protrusion angles  $\theta$  are presented and compared with the numerical results. Our measurements clearly demonstrate the dependence of the effective slip length on both  $\theta$  and  $\phi$ . When the fraction of the surface covered by bubbles is  $\phi = 0.54$ , a maximum effective slip length of  $4.8 \pm 0.1 \mu\text{m}$  was measured at  $\theta = 12^\circ \pm 2^\circ$ . In addition, the experimental results reveal a decrease in the effective slip length with increasing protrusion angles when  $\theta \geq 12^\circ$ . A similar trend was observed for  $\phi = 0.38$ . For larger protrusion angles,  $\theta \geq 10^\circ$ , the measured effective slip length decreases with increasing protrusion angle, implying an increasing flow resistance due to larger protrusions. There is good quantitative agreement with the numerical results.

Our simulations reveal an asymmetry in the effective slip length between positive and negative protrusion angles, affirming previous reports (13, 14, 18–20). Consistent with the experimental results, the numerical  $b_{\text{eff}}$  increases with increasing protrusion angle to a maximum value at  $\theta = 11^\circ$  and subsequently decreases with further increases in  $\theta$  for all  $\phi$ . A peak position of  $b_{\text{eff}}$  was also encountered at a similar  $\theta \approx 10^\circ$  in the theoretical solutions provided in refs. 14 and 18 and highlighted as an optimum angle in ref. 14. The effective slip length becomes zero at a critical protrusion angle,  $\theta_c$ , above which the microbubbles exhibit negative slip length, revealing a transition from slippage to extra friction. Our simulations yield  $\theta_c \approx 62^\circ$  and  $\theta_c \approx 68^\circ$  for  $\phi = 0.54$  and  $\phi = 0.38$ , respectively, which are quantitatively consistent with the results of previous theoretical and numerical studies (13, 14, 18–20).

Our numerical data indicate that for  $\theta$  beyond the value  $\theta = 11^\circ$  giving maximal slip length, higher bubble fractions (here  $\phi = 0.54$ ) follow a steeper decrease in the effective slip length than lower ones (here  $\phi = 0.38$ ).  $b_{\text{eff}}$  is smaller for  $\phi = 0.54$  (with a smaller  $\theta_c \approx 62^\circ$ ) than for  $\phi = 0.38$  when  $\theta \geq 52^\circ$  (Fig. 3A). The increase in protrusion angle alters the flow cross-sectional area. When the typical length scales of the system are comparable with the scale of bubble units, the changes in the flow cross-sectional



**Fig. 2.** Velocity profiles measured by  $\mu$ PIV. (A) Bright-field image of a bubble unit with a protrusion angle of  $\theta = 43^\circ \pm 6^\circ$ . (B) Velocity field measurement for bubbles at  $\theta = 43^\circ \pm 6^\circ$  superimposed on its raw  $\mu$ PIV image. In A and B, the dashed lines represent the bubble curvature and the solid lines represent the no-slip solid walls. (C) Liquid-gas interfaces showing the bubbles having protrusion angles of  $\theta = 43^\circ \pm 6^\circ$ . The curvature of the bubbles is calculated by circular arc estimation. Colored lines indicate the vertical positions of the velocity measurements presented in D. (D) Detailed streamwise velocity profiles,  $u_x(y)$ , at different channel heights  $y$ , indicated by the corresponding dashed lines in C. (E) Average velocity profile for  $\theta = 43^\circ \pm 6^\circ$  (experimental,  $\blacksquare$ ; and numerical,  $\_$ ). (F) Experimental average velocity profiles for  $\theta = 43^\circ \pm 6^\circ$  at a shear-free fraction  $\varphi = 0.38$  ( $\blacksquare$ ) and  $\theta = 21^\circ \pm 3^\circ$  at a shear-free fraction  $\varphi = 0.54$  ( $\bullet$ ), near the wall with attached bubbles. The dashed lines represent the linear fits for the  $b_{eff}$  evaluations.

area and the interface curvature have effects on the velocity field. These observations are consistent with the previous studies, suggesting the significant role of confinement effects on slippage over gas-liquid interfaces longitudinal to flow direction (11, 12).

Our findings suggest that the confinement effects and the gas-liquid interface geometry of the microbubbles have a strong effect on the effective slip length. Furthermore, the experimental results imply the possibility of engineering hydrodynamic slippage/drag by controlling the shape of the gas-liquid meniscus via external means.

To highlight the capability of tailoring the effective boundary condition in our microfluidic devices, we recast the experimental and numerical data in terms of the Fanning friction factor  $C_f$ , which for a fully developed, steady, laminar flow in a smooth rectangular duct of hydraulic radius  $R_h$  is (23)

$$C_f = \frac{R_h \Delta P}{\rho \langle u_x \rangle^2}. \quad [1]$$

Here  $\Delta P$  (Pa/m) is the pressure gradient,  $\rho$  is the liquid density, and  $\langle u_x \rangle$  is the mean velocity. Solving 2D Stokes flow for a rectangular duct using Navier's slip condition at  $y = 0$ , we obtain an expression for the pressure drop along the microchannel length:

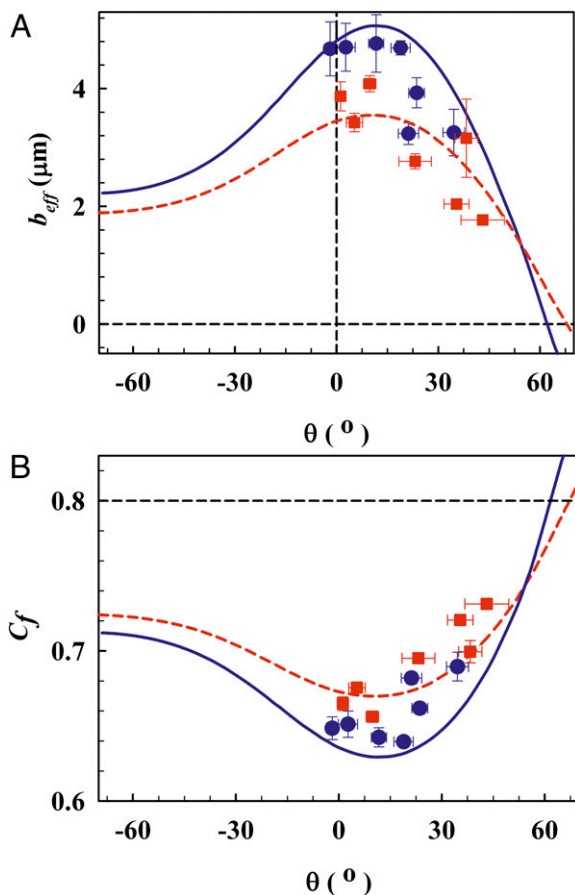
$$\Delta P = \frac{-12\mu \langle u_x \rangle (b_{eff} + H)}{H^2 (4b_{eff} + H)}. \quad [2]$$

Here,  $\mu$  is the dynamic viscosity of the liquid. Using our experimental and numerical results, we calculate  $\Delta P$  from Eq. 2 to evaluate the friction factors for pressure-driven flow of water past microbubbles in our microfluidic devices. In Fig. 3B, we

present the effective friction factor values for varying protrusion angles of microbubbles for  $\varphi = 0.38$  and  $\varphi = 0.54$ .

The friction factor is a direct measure of the pressure loss due to hydrodynamic drag. The effective friction factors of our microbubble mattress should be less than that of a rectangular duct with no-slip walls for  $\theta < \theta_c$ . The analytical model reduces to  $C_f = 10.67/Re$  for a straight rectangular channel ( $b_{eff} = 0$ ) with a width-to-height aspect ratio of 2, corresponding to our microchannels. The analytical  $C_f$  value for  $Re \approx 13$ , as in our  $\mu$ PIV experiments and numerical simulations, is calculated as 0.8 for a nonslippery microchannel. This analytical friction coefficient, depicted by the dashed line in Fig. 3B, is larger than the friction coefficients of our hydrophobic microchannels with embedded microbubbles for  $\theta < \theta_c$ . At a critical  $\theta = \theta_c$ , the numerical results of  $C_f$  are equal to the analytical friction factor ( $C_f = 0.8$ ) for no-slip walls. When  $\theta > \theta_c$ , a higher friction factor appears, revealing a transition from slippage to extra friction. Fig. 3B emphasizes the significant effect of the interface menisci curvature on the hydrodynamic slippage. The numerical results show when  $\theta \approx 11^\circ$ , 18% and 23% drag reductions are achieved for  $\varphi = 0.38$  and  $\varphi = 0.54$ , respectively, compared with the flow in a nonslippery microchannel at the same flow rate. In good agreement with the numerical results, experimental drag reductions of 19% and 21% are obtained for  $\varphi = 0.38$  and  $\varphi = 0.54$ , respectively, when  $\theta$  is in the range of  $-2^\circ$  to  $12^\circ$ .

In conclusion, we present a hydrophobic microfluidic device that allows for the manipulation of flow resistance. The proposed design of the microfluidic device allows for the formation of stable and controllable microbubbles that are perpendicular to the pressure-driven flow in the microchannels. Our experimental measurements, which cover a wide range of protrusion angles, reveal a strong dependence of the effective hydrodynamic slip on the gas-liquid interface curvature. Our experimental results



**Fig. 3.** Effective slip length  $b_{eff}$  and effective friction factor  $C_f$  as a function of the protrusion angle  $\theta$  obtained by  $\mu\text{PIV}$  measurements and numerical calculations. (A) Experimental and numerical  $b_{eff}$  results for  $\varphi = 0.54$  and  $\varphi = 0.38$ . (B) Experimental and numerical  $C_f$  results for  $\varphi = 0.54$  and  $\varphi = 0.38$ . In A and B, the solid line (—) and the circles (●) indicate the numerical and experimental results for  $\varphi = 0.54$ . The dashed line (---) and the squares (■) indicate the numerical and experimental results for  $\varphi = 0.38$ . The horizontal black dashed line represents the value  $C_f = 0.8$  obtained for the no-slip condition  $b = 0$ .

confirm the numerical simulations for effective slip length and the effective friction factor. Our microfluidic device allows for the tuning of the convective flow throughput and, hence, the control of flow resistance. This active control is of paramount importance for microfluidic applications aiming to amplify the transport of fluids and particles at interfaces, which is driven by a convection–diffusion mechanism (24).

### Materials and Methods

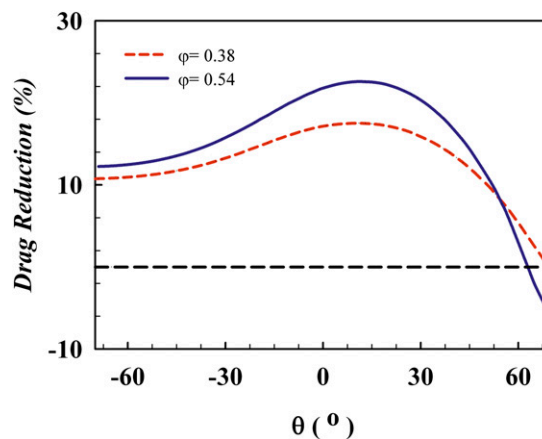
**Experimental Setup.** Silicon microchannels were fabricated by photolithography followed by a deep reactive ion etching process. The microchannels were sealed by anodic bonding to glass. The width of the main microchannels,  $H$ , is  $50\ \mu\text{m}$ . The width of the gas-filled side channels  $L_g$  was kept constant at  $20\ \mu\text{m}$  and the length of the liquid–solid interface  $L_s$  was varied ( $30\ \mu\text{m}$  and  $20\ \mu\text{m}$ , respectively). The periodic bubble unit length is given as  $L = L_g + L_s$ . The shear-free fraction (surface porosity) is defined as  $\varphi = L_g/L$  (Fig. 1C). The ratio of the bubble unit cell length  $L$  to the channel hydraulic radius  $R_h = 33.3\ \mu\text{m}$  is  $\delta = L/R_h$ . The effective slip length was quantified for different shear-free fractions  $\varphi$  ( $\sim 0.38$  and  $\sim 0.54$ ) and spatial periods normalized by the hydraulic radius,  $\delta = L/R_h$  ( $= 1.5$  and  $= 1.2$ ). The protrusion angles of the microbubbles were varied over a wide range ( $-2^\circ$  to  $43^\circ$ ) with small gas pressure changes applied ( $0.29$ – $0.34$  bar) for a liquid flow rate of  $45\ \mu\text{L}/\text{min}$ . We operated at a sufficiently small capillary number,  $Ca = \tau_w/(\sigma/L_g) \approx 5 \times 10^{-3}$ , and Weber number,  $We = (\rho(u_x)^2 L_g)/\sigma \approx 6 \times 10^{-3}$ , to ensure symmetric microbubbles

pinned at the sharp corners of the side channels. Here  $\tau_w$  is the shear stress imposed by the liquid on the hybrid wall and  $\sigma$  is the interfacial tension of the air–water interface.

**$\mu\text{PIV}$ .** Steady, laminar velocity profiles at the focal plane in the middle of the microchannel depth were measured using a  $\mu\text{PIV}$  technique, as described in ref. 11. Milli-Q water seeded with  $1\text{-}\mu\text{m}$  diameter fluorescent particles was used as the working fluid. A dual-cavity Nd:YAG laser at  $532\ \text{nm}$  was used for channel illumination. Image pairs with a delay time of  $7\ \mu\text{s}$  between two exposures were recorded using a double-shutter PCO Sencam camera with a resolution of  $1,376 \times 1,040$  pixels  $\times$  12 bits. To enhance particle visibility and the signal-to-noise ratio of the correlation map, image preprocessing was performed before cross-correlation. Averaged mean intensity images were calculated and subtracted from the image pairs. The particle image density was artificially increased by using a consecutive sum of five images in a row to increase the resolution of the final vector field. The interrogation view of the  $\mu\text{PIV}$  images was  $\sim 222\ \mu\text{m} \times \sim 167\ \mu\text{m}$ . A multigrid ensemble correlation averaging method was used for 195 image pairs. When processing the data, the interrogation window size was decreased in steps to a size of  $32 \times 16$  pixels ( $\sim 5.2\ \mu\text{m} \times 2.6\ \mu\text{m}$ ) to achieve a high spatial resolution for the detailed velocity profiles. Our data have high signal-to-noise ratios. Therefore, no smoothing needed to be applied to the velocity fields. For each measurement, bright-field images were acquired to determine the protrusion angle and to define the locations of the bubbles and walls on the raw  $\mu\text{PIV}$  images.

**Numerical Analysis.** The effective slip length was also numerically calculated using a 2D finite-element method (Comsol Multiphysics v4.1) that solved the Navier–Stokes equations for a steady pressure-driven flow of water in a microchannel consisting of 15 successive bubble units at the bottom surface. A two-bubble-unit cell length was required for the entrance/outlet effects and developing flow effects. The bubbles are pinned and approximated as rigid circular arcs calculated by the projected diameter of bubble,  $L_g$ , and the protrusion angle  $\theta$ . The bubble interface curvature is parameterized by the protrusion angle  $\theta$ . A perfect slip boundary condition for the bubble surfaces and a no-slip boundary condition for solid walls were assumed. The upper solid wall is a nonslipping wall at a distance  $y = H$ . Pressure-driven flow was produced by applying a laminar flow with a mean velocity of  $0.2\ \text{m/s}$  as the inlet condition. The effective slip length for all  $\theta$  is calculated at an evaluation line,  $y_e = 6.5\ \mu\text{m}$ , due to the protrusion depth of bubbles in the  $y$  direction into the channel. The evaluation line at height  $y_e$  is sufficiently above the bubble surfaces for all  $\theta$ . The effective slip length is calculated on the basis of Navier’s slip boundary condition. The ratio of the velocity  $u_x(y_e)$  to the tangential shear rate at  $y_e$ ,  $du_x/dy$ , is integrated along  $x$  over the middle 11 bubble units, and  $y_e$  is subtracted from the resultant value to obtain  $b_{eff}$ .

**Derivation.** To obtain the Fanning friction factor  $C_f$ , the 2D, steady, fully developed Stokes flow between parallel plates was analytically solved using



**Fig. 4.** Drag reduction as function of the protrusion angle  $\theta$  obtained by simulations. The solid line (—) and the dashed line (---) represent the results for  $\varphi = 0.54$  and  $\varphi = 0.38$ , respectively. The horizontal black dashed line represents the no-slip condition ( $b = 0$  and  $C_f = 0.8$ ).

**Table 1. Comparison of effective slip length results with the asymptotic results of ref. 5**

$\varphi$	$\delta$	$b_{eff}$				
		$\mu$ PIV	Numerical	$b_{AL1}$	$b_{AL2}$	$b_{AL3}$
0.38	1.5	$3.9 \pm 0.3$	3.5	3.2	1.5	5.1
0.54	1.2	$4.7 \pm 0.5$	4.9	4.5	2.6	9.8

All slip length values are in micrometers. The last three columns refer to the values obtained from the asymptotic expressions given in Eqs. 4a–4c.

the no-slip boundary condition at the upper plate,  $y=H$ , and the full-slip boundary condition at the bottom plate,  $y=0$ . Due to the protrusion depth of bubbles in the  $y$  direction, the effective hydrodynamic boundary position depends on  $y$ . We incorporate the effective slip length  $b_{eff}$ , using a linear approximation of velocity in the close proximity of the solid wall,  $y=y_e$ , yielding the following relation between the slip velocity at  $y=0$  and the velocity at  $y=y_e$ :

$$u_x(y=0) = u_x(y=y_e) \frac{b_{eff}}{b_{eff} + y_e} \quad [3]$$

Thereby,  $u_x(y=y_e)$  can be expressed as  $(b_{eff} + y_e)(du_x/dy)|_{(y=0)}$ , and the pressure drop can be expressed in terms of  $b_{eff}$  (Eq. 2). For a fully developed, steady laminar flow, the shear stress is given as  $\tau_w = R_h \Delta P / 2$ . The definition of the Fanning friction factor  $C_f = \tau_w / (0.5 \rho \langle u_x \rangle^2)$  then immediately yields Eq. 1. It is worth noting that for the case of a straight rectangular duct ( $b_{eff} = 0$ ), Eq. 2 reduces to plane Poiseuille flow  $\Delta P = (-12 \mu \langle u_x \rangle) / H^2$ , which can be further used in Eq. 1 to derive  $C_f = (12 \mu R_h) / (\rho \langle u_x \rangle H^2)$ . For a rectangular duct with an aspect ratio of 2, corresponding to our microchannels,  $R_h = 2H/3$ , yielding  $C_f = 10.67 / Re$  for a smooth, nonslippery rectangular duct. Hydrodynamic drag reductions can be calculated for different protrusion angles in comparison with nonslippery microchannels. In Fig. 4, we show the extent of drag reduction that can be tuned by controlling the shape of the gas–liquid menisci curvature. Here the black dashed line represents a microchannel with nonslippery walls ( $b_{eff} = 0$  and  $C_f = 0$ ).

## Appendix

To further validate our experimental and numerical results for  $b_{eff}$  presented in Fig. 3A, we compare these results with the

analytical asymptotic solutions of ref. 5. Three distinct asymptotic limits for  $b_{eff}$  are considered, as suggested in ref. 5:

$$AL1: b_{AL1} \sim \frac{\varphi}{4} R_h \quad \varphi \ll 1, \quad \delta \text{ fixed} \quad [4a]$$

$$AL2: b_{AL2} \sim \frac{\delta}{2\pi} \ln \left( \sec \left( \varphi \frac{\pi}{2} \right) \right) R_h \quad \delta \rightarrow 0, \quad \varphi \text{ fixed} \quad [4b]$$

$$AL3: b_{AL3} \sim \frac{\varphi}{4(1-\varphi)} R_h \quad \delta \rightarrow \infty, \quad \varphi \text{ fixed}, \quad [4c]$$

where  $R_h$  is the hydraulic radius, and  $\delta = L/R_h$ . The first asymptotic limit (Eq. 4a) describes the limit at which the bubble fraction goes to zero for a given ratio of bubble unit length to hydraulic radius. The second (Eq. 4b) and the third (Eq. 4c) asymptotes describe the limits at which  $\delta$  goes to zero and infinity, respectively, for a given  $\varphi$ . In Table 1, we compare our numerical  $b_{eff}$  values obtained at  $\theta = 0^\circ$  and our experimental  $b_{eff}$  values measured at  $\theta = 1.2^\circ \pm 0.3^\circ$  and  $\theta = -1.9^\circ \pm 0.5^\circ$  for  $\varphi = 0.38$  and  $0.54$ , respectively, with the predictions resulting from Eqs. 4a–4c. Indeed, the results of effective slip length obtained from both the  $\mu$ PIV measurements and the numerical simulations agree well with the asymptotic prediction of the analytical solution, Eq. 4a. The slight discrepancy can be explained by the asymptotic limit of small  $\varphi$  for a constant  $\delta$  of  $b_{AL1}$ . The asymptotic solutions  $b_{AL2}$  and  $b_{AL3}$  refer to the minimum and maximum slip lengths for the asymptotic extremes in  $\delta$ . Table 1 indicates that for all  $\varphi$ ,  $b_{AL2} < b_{eff} < b_{AL3}$ , which further validates the consistency of our experimental and numerical effective slip length results evaluated for an intermediate  $\delta$ .

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