Using the coupled wake boundary layer model to evaluate the effect of turbulence intensity on wind farm performance

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Abstract. We use the recently introduced coupled wake boundary layer (CWBL) model to predict the effect of turbulence intensity on the performance of a wind farm. The CWBL model combines a standard wake model with a “top-down” approach to get improved predictions for the power output compared to a stand-alone wake model. Here we compare the CWBL model results for different turbulence intensities with the Horns Rev field measurements by Hansen et al., Wind Energy 15, 183196 (2012). We show that the main trends as function of the turbulence intensity are captured very well by the model and discuss differences between the field measurements and model results based on comparisons with LES results from Wu and Porté-Agel, Renewable Energy 75, 945-955 (2015).

1. Introduction
For the design of wind farms it is important to understand the effect of the relative turbine positioning on the overall power output [1]. Two analytical approaches have been used for such evaluations. The first approach, is the use of classical wake models [2–8] to estimate the wind farm performance. Such models have been used in several wind farm optimization studies [9–12]. Wake models predict the wake deficits in the entrance region of the wind farm well, but have difficulty predicting the performance further downstream where many wakes interact [13–15]. “Top-down” models [16–18] can capture the vertical structure of the atmospheric boundary layer and the associated wake-atmosphere interactions in this fully developed regime of the wind farm better than wake models. Therefore “top-down” models have been used for the evaluation of the optimal spacing in very large wind farms [19, 20]. However, “top-down” models do not consider the effect of the relative turbine positioning [21, 22] on the wind farm performance. Previous works have used a one-way coupling between wake and “top-down” models [6, 17, 23] to improve the performance.

In this paper we use the recently introduced coupled wake boundary layer (CWBL) model [15,22] to predict the influence of the turbulence intensity on the performance of Horns Rev. In particular, to investigate how well computationally inexpensive analytical models can predict the influence of this flow feature. The CWBL model combines the Calaf et al. [18,24] “top-down” model and a classical wake model [3, 4, 6] through two-way coupling. The effect of the
relative turbine positioning is captured by the wake model part and the interaction between the wind farm and the atmospheric boundary layer by the “top-down” part. It has been shown that the CWBL model [15, 22] gives improved predictions over either of its constitutive parts for the turbine power production in the fully developed regime of the wind farm. In this proceeding we start with a very short review of the basic concepts of the CWBL model in section 2. In section 3 the CWBL model results for different turbulence intensities are compared with field measurement data for Horns Rev [25] and corresponding LES results by Wu and Porté-Agel [26]. Section 4 concludes the paper.

2. The generalized CWBL model

In the CWBL model both the wake (section 2.1) and the “top-down” (section 2.2) model have one parameter that needs to be determined. An iterative procedure (section 2.3) is used to calculate these parameters from the complementary part of the CWBL model to make sure that a consistent CWBL system is obtained. For a detailed description of the (generalized) CWBL model we refer the reader to Refs. [15, 22].

2.1. Wake model part

The classic wake model [2–4, 6, 7] assumes that wind turbine wakes grow linearly with downstream distance and uses the following expression for the velocity (which is simplified as a top-hat distribution) in the turbine wakes [2, 3, 5]:

\[ u = u_0 \left( 1 - \frac{1 - \sqrt{1 - C_T}}{1 + k_w x/R} \right) = u_0 \left( 1 - \frac{2a}{(1 + k_w x/R)^2} \right). \]  

(1)

Here \( u_0 \) is the incoming free stream velocity, \( k_w \) is the wake expansion coefficient, \( R \) is the rotor radius, \( C_T = 4a(1 - a) \) is the thrust coefficient with flow induction factor \( a \), and \( x \) is the downstream distance with respect to the turbine, which is modeled as an actuator disk. Wake interactions are accounted for by adding the squared velocity deficits of interacting wakes [4]. The predicted normalized turbine power \( P_T/P_1 \) is given by

\[ \frac{P_T}{P_1} = \left( \frac{1}{N_d} \sum_{k=1}^{N_d} \frac{u(x_{T,k})}{u_0} \right)^3. \]  

(2)

Here the summation \( k \) is over all points \((N_d)\) on the turbine disk and the velocity ratio is obtained by calculating the effect of all wind turbine wakes, including the wake interaction effects, at these locations. We use a uniform inflow, which is common in wake models. Computing the sum over the entire disk, instead of just considering the value at hubheight, ensures that the ground effect modeled by the image turbines sets in incrementally, i.e. there are no discontinuous changes in the wake behavior with increasing the streamwise distance.

The wake decay parameter at the entrance of the wind farm \( k_{w,0} \) can be modeled as the ratio of friction velocity to the mean velocity at hubheight, which gives

\[ k_{w,0} = \frac{\kappa}{\ln(z_h/z_{0,lo})}. \]  

(3)

where \( z_{0,lo} \) is the roughness length of the ground surface, \( z_h \) the turbine hubheight, and \( \kappa = 0.4 \) is the von Kármán constant [2, 17, 27]. To match the wake expansion coefficient to the turbulence intensity, expressed based on the streamwise velocity fluctuations, we use the following logarithmic laws for the mean

\[ \langle \nu \rangle / u_s = \kappa^{-1} \ln(z/z_{0,lo}) \]  

(4)
Figure 1. (a) The turbulence intensity at hubheight as a function of the wake expansion coefficient at the entrance of the wind-farm, $k_{w,0}$. The data points give the recommended relationship by Windpro [5], which is based on comparisons with field measurement data. The black line is based on the model equations (3), (6), and (7). The turbulence intensity as function of (b) the roughness length $z_0$, $lo$, and as function of (c) the boundary layer height for the data shown in panel (a).

and variance

$$\langle(u'^2)\rangle = B_1 - A_1 \log(z/\delta),$$

in the boundary layer [28–30].

Equation (4) gives the mean velocity profile in an atmospheric boundary layer, while equation (5) gives the streamwise velocity fluctuations in the boundary layer. Combining both equations gives the turbulence intensity at hubheight as

$$T_{hub} = \left(\frac{\langle(u'^2)\rangle^{1/2}}{\langle u \rangle}\right) = \left[\frac{B_1 - A_1 \log(z_h/\delta)}{\kappa^{-1} \ln(z_h/z_{0,lo})}\right]^{1/2}.$$

The constants $A_1$ and $B_1$, are based on those measured in different high Reynolds number turbulent boundary layer experiments, see Marusic et al. [28] for an overview. They concluded that $A_1 \approx 1.25$ is an universal constant, while $B_1 \approx 1.5 - 2.1$ depends slightly on the flow geometry. Based on comparisons of our atmospheric boundary layer simulations with experiments we concluded that $B_1 \approx 1.6$ is an appropriate value for this case [30]. According to equation (6) the turbulence intensity at hubheight depends on the roughness length $z_{0,lo}$ and the boundary layer height $\delta$, while equation (3) states that the wake expansion coefficient at hubheight only depends on $z_{0,lo}$.

In addition, we use that the atmospheric boundary layer height is given by [31]

$$\delta = C u_*/f$$

where $f = 2\Omega \sin(\psi)$ with $\Omega = 2\pi/(24 \times 3600s) = 7.27 \times 10^{-5}$ 1/s, $\psi = 55^\circ$ (the latitude of Horns Rev), and $C = 0.15$, which is typical for neutral atmospheric boundary layers [32–34]. We note that stratification affects the very largest scales the most, i.e. things happening at a height $z = \delta$, but that these effects are much weaker at hubheight, and below (i.e. where $z_{0,lo}$ enters). We verified in the model calculations that small changes in $C$ do not significantly influence the results. Using equations (3), (6), and (7) we obtain for each turbulence intensity the corresponding boundary layer height $\delta$, the roughness height of the ground $z_{0,lo}$, and wake expansion coefficient $k_{w,0}$. Figure 1 shows that the results agree very well with the recommended values by Windpro [5], which are based on comparisons with field measurement data.
2.2. The “top-down” model
In the CWBL model we use the “top-down” approach introduced by Calaf et al. [18]. This model is used to obtain the ratio of the mean velocity in the fully develop regime to the reference incoming velocity at hubheight
\[
\frac{\langle u \rangle (z_h)}{\langle u_0 \rangle (z_h)} = \frac{\ln (\delta_{IBL}/z_{0,lo})}{\ln (\delta_{IBL}/z_{0,hi})} \ln \left[ \left( \frac{z_h}{z_{0,hi}} \right) \left( 1 + \frac{D}{2z_h} \right) \right]^{\beta} \left[ \ln \left( \frac{z_h}{z_{0,lo}} \right) \right]^{-1}.
\]  
(8)

Here \(\delta_{IBL}\) indicates the height of the internal boundary layer in the fully developed regime of the wind farm, \(D\) the turbine diameter, \(\beta = \nu_w^*/(1 + \nu_w^*)\) and \(\nu_w^* \approx 28\sqrt{\pi C_T/(8s_x s_{ye})}\), and \(z_{0,hi}\) denotes the roughness length of the wind farm, which is defined as
\[
z_{0,hi} = z_h \left( 1 + \frac{D}{2z_h} \right)^\beta \exp \left( - \left[ \frac{\pi C_T}{8s_x s_{ye} \kappa^2} + \left( \ln \left( \frac{z_h}{z_{0,lo}} \left( 1 - \frac{D}{2z_h} \right) \right) \right]^{-2} \right]^{-1/2} \right).
\]  
(9)

Here \(s_{ye}\) is the effective spanwise turbine spacing, which is obtained from the two-way coupling with the wake model part of the CWBL model. Due to the two-way coupling between the wake and the “top-down” model \(s_{ye}\) depends on parameters such as the streamwise distance between the turbines \(s_x\), the relative positioning of the turbines, and the wake coefficient in the fully developed regime of the wind farm \(k_{w,\infty}\).

The reason for changing the effective spanwise spacing \(s_{ye}\) in the “top-down” model is that this model considers a momentum balance averaged over the entire horizontal plane. In that model the horizontally averaged velocity thus depends on the friction velocity, which depends on the stresses that are imposed by the turbines in the wind farm. However, when the spanwise spacing between the turbines is very large this assumption is no longer valid. This fact can be clearly seen from cases with very small streamwise spacing, \(s_x\), and very large spanwise spacings, \(s_y\), which in the “top-down” model (that only depends on \(s\)) will result in limited wake effects. This assumption is obviously unrealistic, as even for a single line of turbines aligned in the wind direction for which \(s_y = \infty\) a significant power reduction is observed for downstream turbines. In the CWBL model we assume that the momentum analysis in the “top-down” part of the model should be performed over the control area that is representative of the region that is directly influenced by the wind turbine wakes. Since the wakes progress very far in the downstream direction, the streamwise spacing, \(s_x\), is unadjusted in the model, while the wake expansion is limited in the spanwise direction. In the CWBL modeling approach the effective spanwise spacing, \(s_{ye}\), that should be used is obtained from a two-way coupling procedure with a wake model. For further details about this procedure we refer the reader to section IV of Ref. [15].

2.3. Coupling
The wake and “top-down” parts of the CWBL model are coupled by demanding that both models give the same prediction for the turbine velocity in the fully developed regime of the wind farm. An iterative procedure is used to obtain the effective spanwise spacing, \(s_{ye}\), in equation (9) and the wake expansion rate, \(k_{w,\infty}\), in equation (1) in the fully developed regime. The effective spanwise spacing, \(s_{ye}\), is the spanwise dimension of the control volume size that is used to account for the large scale interactions with the atmosphere as explained above. Because in the wake model the turbine velocity in the fully developed regime depends on \(k_{w,\infty}\), while in the “top-down” model this velocity depends on \(s_{ye}\), these values need to be iterated until convergence is reached and this iteration is accomplished through the two-way coupling in the CWBL model. This procedure is described in detail for aligned and staggered wind farms in Stevens et al. [15]. To make sure that the effect of the wind farm geometry is taken into account...
this two-way coupling should be enforced for each wind direction separately, which is described in detail in Ref. [22]. Here we in addition, iterate the internal boundary layer thickness in the fully developed regime as

$$\delta_{IBL} = \delta_{u_{hi}} u^*_{hi,},$$

where

$$\frac{u_{hi}}{u^*_{hi}} = \frac{\ln (\delta_{IBL}/z_{0,lo})}{\ln (\delta_{IBL}/z_{0,hi})},$$

is obtained from the CWBL model. We limit the internal boundary layer thickness to 500 meters when we compare with the LES data (as this was the boundary layer thickness used in the corresponding simulations by Porté-Agel *et al.* [35]) and 1000 meters (height of the thermal inversion) for the comparison with the field measurements.

In order to capture the entrance effects the CWBL model assigns a wake coefficient

$$k_{w,T} = k_{w,\infty} + (k_{w,0} - k_{w,\infty}) \exp(-\zeta m),$$

(12)

to each turbine in the wind farm by interpolating between $k_{w,0}$ (the wake expansion coefficient at the entrance of the wind farm) and $k_{w,\infty}$ (the wake expansion coefficient in the fully developed regime, which is found using the iterative procedure). Here $\zeta = 1$ and $m$ is the number of turbine wakes that interact with the turbine of interest. Thus the wake model part dominates in the entrance region of the wind farm, while the wake development further downstream is determined by the two-way coupling between the wake and “top-down” models that comprise the CWBL model. Therefore the CWBL predictions in the fully developed regime of the wind farm depend much less on the quadratic superposition of the wakes [4] than those of the stand alone wake model.

In this proceeding we assumed that all turbines operate with the same thrust coefficient $C_T$. In the field measurements [25] and the LES results [26] to which we compare the model predictions the wind speeds considered correspond to turbines operating in region II for which this assumption is reasonable. Because the measurement consider a very narrow range of wind speeds we neglected the variation of the power coefficient $C_P$ with wind speed as this effect cancels out when relative powers are considered.

3. Results

Figure 2a shows the layout of the Horns Rev wind farm, which consists of 80 turbines with a hubheight of 70 meters and a turbine diameter of 80 meters, in a rectangular pattern. The streamwise spacing (East-West direction) is 7 turbine diameters and the spanwise spacing (North-South) is 6.95 turbine diameters. Figure 2 shows the power deficits as function of the downstream distance averaged for the wind directions $270^\circ \pm 15^\circ$, $222^\circ \pm 15^\circ$, and $312^\circ \pm 15^\circ$, and a wind speed of $8 \pm 0.5$ m/s with an average turbulence intensity of $7.0\%$ ($6.3\%$ for $222^\circ \pm 15^\circ$) (see figure 4 of Hansen *et al.* [25]). The corresponding model results capture these measurements quite accurately. The figure also shows that the CWBL model agrees well with LES results from Wu and Porté-Agel [26].

The study of Hansen *et al.* [25] also analyzed the effect of turbulence intensity and atmospheric stability on the downstream power development. Hansen *et al.* [25] defined different stability classes ($cL = 3$, $cL = 2$, and $cL \leq 1$) based on the Obukhov length, see table 1 of Ref. [25]. In the remainder of the proceeding we will refer to these different cases using the naming convention introduced by Hansen *et al.* [25], i.e. “very stable” (case label $cL = 3$), “stable” (case label $cL = 2$), and “remaining” (case label $cL \leq 1$). From figure 7 of their paper we obtain that the average turbulence intensity of these three main stability classes they consider are $7.1\%$, $5.1\%$, and $3.9\%$, respectively, when the wind speed is $8 \pm 0.5$ m/s. We use these reported
turbulence intensities to distinguish the different atmospheric stability classes. We note that the effect of the atmospheric stability is not included in the generalized CWBL model, but that the effect of increasing turbulence intensity is included. Figure 3a shows the measured power deficits for these three stability classes as a function of the downstream direction for the $270^\circ \pm 15^\circ$ direction. Panel (b) of that figure shows the model results for the three corresponding turbulence intensities. The lower panels of figure 3 compare the field experiments and the model results for the three cases separately. In agreement with the experimental observations, the model predicts an increased velocity deficit for decreasing turbulence intensity. The reason is that a higher turbulence intensity results in a wake that better mixes with the surrounding flow. The figure shows that the effect of the turbulence intensity on the wind turbine performance is more pronounced further inside the wind farm.
Figure 3. The power deficit along rows of wind turbines with a streamwise spacing of 7D averaged for the wind directions 270±15° and a wind speed of 8±0.5 m/s. The field measurement results are grouped in the very stable (cL = 3), stable (cL = 2) and remaining (cL ≤ 1) stability classes. The CWBL predictions have been computed for three different turbulence intensities, i.e. 7.1%, 5.1%, and 3.9%, which correspond to the average turbulence intensity for these three different stability classes. Panels (a) and (b) show a comparison of the three different cases for the field measurements and the CWBL model separately, and panels (c), (d), and (e) compare the field measurements and the corresponding CWBL predictions. The data from Hansen et al. [25] are digitally extracted.

Figure 4 shows the same results as in figure 3 for the wind-directions 222±15° and 312±15°. A comparison with figure 3 reveals that the effect of changes in the turbulence intensity result in similar changes in the wind farm performance for the different wind directions. Considering that the effective streamwise distance is “similar” for these different wind directions (222°, 270°, 312°), i.e. 9.4D, 7.0D, and 10.4D, respectively, and the overall wind turbine density is the same for these cases, this is reasonable. However, the corresponding experimental data seem to indicate a different trend, specifically a bigger influence of the turbulence intensity on the performance for the 222 ±15° and 312 ±15° cases than for the 270 ±15°. We do not know the reason for this discrepancy. However, we do note that it is well known in the literature that the comparison with the Horns Rev data to model results tends to be more favorable for some wind-directions than for others [36].

Here we also emphasize that, apart from the turbulence intensity that is included in the model, the effect of the atmospheric stability is not accounted for in the model. So, for example, the effect of the atmospheric thermal stability on the wind shear is not taken into account. Generalizations to this approach can be made by including stability correction functions. Such a direction has been explored in a recent paper dealing with another topic (Sescu & Meneveau...
Figure 4. The power deficit for the wind directions $222^\circ \pm 15^\circ$ (top panels) and $312^\circ \pm 15^\circ$ (lower panels) for a wind speed of $8 \pm 0.5$ m/s. The field measurement results are grouped in the very stable ($cL = 3$), stable ($cL = 2$) and remaining ($cL \leq 1$) stability classes. The CWBL predictions have been computed for three different turbulence intensities, i.e. 7.1%, 5.1%, and 3.9%, which correspond to the average turbulence intensity for these three different stability classes. The left panels indicate the field measurement by Hansen et al. [25] and the right panels the CWBL predictions. The data from Hansen et al. [25] are digitally extracted.

Nevertheless the CWBL model is able to predict some of the main trends observed in the field data, which seems to indicates that the change of the turbulence intensity due to the thermal effects is one of the effects that influences the wind farm performance. In the field measurements the effect of the different atmospheric conditions seems stronger for some wind directions.

4. Summary
This paper demonstrates that the predicted performance trends by the coupled wake boundary layer (CWBL) model as a function of the turbulence intensity compare well with trends observed in field measurement data of Horns Rev [25]. The model has been compared with the field experiments using the reported turbulence intensities for the different atmospheric stability classes. We find that the model captures the trend that an increase in the turbulence intensity leads to decreased wake defect velocities due to enhanced mixing. The effect of the turbulence intensity is bigger further downstream. In the model predictions the change in the wind
farm performance due to changes in the turbulence intensity are similar for different wind directions. Considering the similarity between these cases this is reasonable. However, in the field measurements the effect of the different atmospheric conditions seems to be stronger for some wind directions. Further research will be necessary to clarify this behavior. Further research will be necessary to clarify this behavior.

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