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**On weakly confluent monadic string-rewriting systems.
(English. English summary)**

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Let Σ be an alphabet provided with a total ordering $>$ and Σ^* the set of words over Σ. A monadic string-rewriting system R is a subset of $\Sigma^* \times \Sigma^*$ of which each rule $l \to r$ in R satisfies $l >_{ll} r$ and $r \in \Sigma \cup \{e\}$. Here e denotes the empty word and $>_{ll}$ the length-lexicographical ordering on Σ^*, i.e., $l >_{ll} r$ iff either $|l| > |r|$, or $|l| = |r|$ and $l >_{\rm lex} r$, where $>_{\rm lex}$ is the pure lexicographical ordering on Σ^* induced by $>$.

Applying rules from R gives rise to a derivation relation of which the reflexive, symmetric, and transitive closure is a congruence on Σ^* with respect to concatenation. R is called weakly confluent if it is confluent on all the congruence classes $[\alpha]_R$ with $\alpha \in \Sigma \cup \{e\}$. Deciding whether a finite monadic string-rewriting system is weakly confluent, turns out to be "tractable" (possible in polynomial time). But many decision problems (e.g., word problem, free submonoid problem) that are tractable for finite, monadic, and confluent systems, are undecidable for finite monadic systems that are only weakly confluent.

However, for finite, monadic, and weakly confluent systems that present groups, the validation problem for linear sentences (a special kind of first-order formulae) is decidable. Several decision problems—e.g., the word problem and the generalized word problem—can be formulated in terms of linear sentences and, consequently, they are decidable in this particular case. Finally, the authors present a specialized completion procedure that, given a finite monadic string-rewriting system presenting a group as input, attempts to construct an equivalent monadic system that is weakly confluent. This procedure consists of two parts: one to delete superfluous rules in order to keep the system reduced, and another to introduce new rules to make the system confluent on the relevant congruence classes. The correctness and completeness of this procedure are shown and some detailed examples are given.

{For the entire collection see MR1218902 (93k:68005)}

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