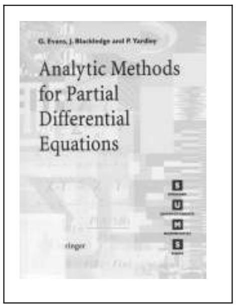
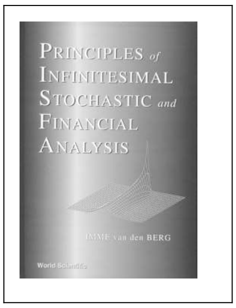
den opgestoken. De auteur moet wel een optimistisch karakter hebben, gezien de zwaarte van nogal wat opgaven. Een rijtje van zeven opgaven eindigt met de vraag om de stelling van Krein-Mil'man te bewijzen. (Een begrensde zwak-∗-gesloten convexe verzameling in de duale van een Banachruimte is het zwak-∗-gesloten convexe omhulsel van haar extreme punten.) De hoofdstukken eindigen met duidelijke opmerkingen over literatuur, alternatieve bewijzen en toepassingen. Helaas zijn deze opmerkingen van voor 1971. Er staan veel opgaven in het boek en gelukkig zijn de moeilijke met een ∗ gemarkeerd. *F.J.L. Martens*



G. Evans, J. Blackledge and P. Yardly
Analytic methods for partial differential equations & Numerical methods for partial differential equations
(Springer Undergraduate Mathematics Series)
London: Springer-Verlag, 2000
299 p., prijs DM 59,- en 290 p., prijs DM 59,-
ISBN 3-540-76124-1 en ISBN 3-540-76125-X

Both books are based on courses given by the authors at De Montfort University in Leicester. The book on analytic methods consists of five chapters containing the following subjects: the method of separation of variables for wave, heat and Laplace equations, first order equations, and hyperbolic second order equations, the Laplace transform method, the Fourier transform method, and Green's functions. The book on numerical methods consists of six chapters in which the following subjects are treated: finite differences, parabolic, hyperbolic and elliptic equations, finite element methods for ODEs, and finite elements for PDEs. Almost all sections in both books have a set of exercises, and the solutions of these exercises can be found in the appendices of both books.

All subjects are treated clearly, and emphasis is put on how to find solutions analytically, or on how to construct approximations of the solutions numerically. These books are suitable for undergraduate students (with some knowledge of ODEs) who want to have an elementary introduction to analytical and numerical methods for partial differential equations. *W.T. van Horssen*



I. van der Berg
Principles of infinitesimal stochastic and financial analysis
Londen: World Scientific Publishing, 2000
136 p., prijs £21,–
ISBN 981-02-4358-8

When teaching a course on option pricing theory one always faces the challenge on how to present the theory to the students. In particular, how to present a good derivation of the most celebrated formula from financial theory: the Black-Scholes formula.

There are two approaches that are usually taken. The first one starts off in discrete time where one can postulate an extremely simple model for the development of asset prices: the binomial tree. The mathematics involved in the binomial model is very basic, only a few linear algebra results lead to a pricing formula for options. Also the basic idea of option pricing, building a replication portfolio, can be made very clear. Unfortunately the final pricing formula expresses the option price in terms of binomial distribution functions and only a limit argument leads then to the Black-Scholes formula.

The other approach is to follow the original Black-Scholes-Merton derivation more closely and to use stochastic processes and Ito calculus. This has the disadvantage that one first needs to build up the machinery of Brownian Motion and Itôcalculus, which many students find very abstract. Furthermore, a fully rigorous build-up can only be based on advanced measure theory. As a consequence most of the time one ends up doing the proof 'by hand waiving' and the derivation of the Black-Scholes formula is then more or less based on faith.

Imme van den Berg's book bridges this gap by using non-standard analysis. For me this was the first time I had ever heard of non-standard analysis, and I am very impressed by the power of this approach. Many of the problems which arise when trying to make the transition from discrete to continuous time stochastic processes are avoided when using the approach of 'an unlimited number of infinitesimal steps' which is formalized by non-standard analysis. Starting from the intuitively appealing setting of a binomial tree, the non-standard analysis approach very naturally derives the well-known continuous-time formulae. Also the proofs are intuitively very appealing: all the technical burden of ?-algebra's, filtrations and 'almost surely' arguments are completely avoided.

If I have to make a critical remark, it is that the book leaves the reader with the impression that it was finished under great time-pressure. Many typos and notational inconsistencies are strewn throughout the book. Furthermore, many of the basic concepts of non-standard analysis like limited, unlimited, appreciable and the notation ? are never properly defined. Their meaning becomes clear from the context, but a formal definition and a more elaborate introduction to non-standard analysis would make the text much more self-contained.

Despite these shortcomings, I would definitely recommend the book to anyone who wants to obtain a better intuitive understanding of the continuous-time results of option-pricing theory. *A. Pelsser*

V. Korotkich
A mathematical structure for emergent computation
(Nonconvex Optimization and its Applications; 36)
Dordrecht: Kluwer, 1999
164 p., prijs NLG 160,–
ISBN 0-7923-6010-9

This monograph, based on six recent papers by the author, deals with a mathematical structure, called integer code series, which is a coding for piecewise constant functions on the integers. Then another structure, called web of relations, is introduced. The author claims that this approach is suitable for describing certain phenomena mathematically as a whole. Within this 'holistic' setting, certain algebraic and geometric observations are made, issues like integer sequences (Prouhet-Thue-Morse, Fibonacci), emergent computations and dynamical systems (chaos, period doubling) are addressed, and a notion of structural complexity is given.

I am afraid that the author's approach is rather isolated and that this monograph will probably not change that situation. This book is anything but a pleasure to read: it is often vague, poorly written and employs incorrect formulations. A typical example, taken from p. viii: "[The author] started to study questions concerning universal principles of emergent computation as a reaction to the general realization that the NP-complete problem probably could not be practically solved by using the Turing model of computation." It also contains overloaded notation (apart from the structural complexity $C(s)$ there is also a function $C(s, s')$ and even a function $C(s(i), s'(i), i)$) and it has no index. The order of the references is not alphabetical and although this volume deals with structural or descriptive complexity, it does not refer to the standard text by M. Li & P.M.B. Vitányi, *An Introduction to Kolmogorov Complexity and Its Applications* (1993), Springer-Verlag. What this monograph has to do with Nonconvex Optimization remains an open question.

In short, I cannot recommend this book to anybody. *P.R.J. Asveld*

N.A. Bobylev, S.V. Emel'yanov et al.

Geometrical methods in variational problems

(Mathematics and its Applications; 485)
Dordrecht: Kluwer, 1999
539 p., prijs NLG 398,–
ISBN 0-7923-5780-9

This monograph is devoted to the study of nonlinear variational problems. The aim of the authors is to provide a self-contained book, accessible to anyone with a basic background in mathematics. For this reason, the authors start with an introductory chapter on functional analysis, going from metric spaces to classical applications of the contraction mapping principle.

The second chapter is devoted to the minimization of nonlinear functionals. After treating the smooth case, the authors consider convex and Lipschitzian functionals, with the corresponding generalized gradients, and Ekeland's variational principle. Interesting applications are given.

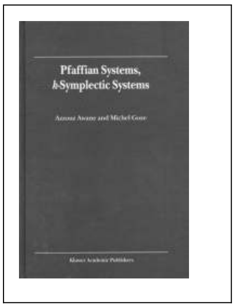
Chapter three deals with a homotopic (or deformation) technique for the study of variational problems. It exhibits a class of functionals and of deformations of such functionals, having each a unique critical point, such that the local minimum character of this point is preserved during the deformation. Applications are given to integral functionals and nonlinear programming problems.

Chapter four considers the characterization of extremals of variational problems using topological degree techniques. Degree theory is presented, using a differential topological approach, in finite dimension, and then extended to completely continuous perturbations of identity, and to some monotone-like mappings in Banach spaces. This allows one to introduce and to compute the topological index of a point of minimum and of an isolated critical set of a functional. Applications are given to problems of the classical calculus of variations and to optimal control theory. The chapter ends with a short introduction to Lyusternik-Schnirel'man minimax theory.

Chapter five is devoted to various applications of the methods developed in previous chapters. It starts with existence theorems for monotone-like gradient operators, applied to nonlinear elliptic boundary value problems, Hammerstein integral equations, problems of elasto-plasticity, and Ginzburg-Landau equations. The mountain pass theorem is then proved and applied to various partial differential equations and ordinary differential systems. Various important inequalities are proved through the homotopic deformation method. A general scheme is presented for the study of degenerate extremals of variational problems (through a finite-dimensional reduction). One then finds infinite-dimensional versions of the Morse lemma, a study of the well-posedness of variational problems, and a thorough exposition of the gradient method for minimizing nonlinear functionals. The remainder of the chapter is devoted to the bifurcation of extremals of variational problems and the existence of eigenvectors for potential operators.

The bibliographical comments are collected at the end of the volume, and refer to a bibliography of more than six hundred items. An index makes easier the access to the rich material contained in this monograph. The material presentation of the volume is good.

By giving an easy and systematic access to a number of techniques recently developed in the former Soviet Union, this book is a very useful and valuable addition to the literature devoted to the modern theory of variational problems. *J. Mawhin*



A. Awane and M. Goze

Pfaffian systems, k-symplectic systems

Dordrecht: Kluwer, 2000
260 p., prijs $105
ISBN 0-7923-6373-6

Een uitwendig differentiaalsysteem is een meetkundige beschrijving van een systeem van partiële differentiaalvergelijkingen, namelijk als een ideaal in de uitwendige algebra van een gladde variëteit. Een belangrijk speciaal geval zijn de systemen van Pfaff, die lokaal voortgebracht worden door vormen van graad 1, hetgeen correspondeert met een systeem van vergelijkingen van de eerste orde. De classificatie van deze systemen was een populaire bezigheid in de decennia rond de vorige eeuwwisseling. Twee uitersten in deze classificatie zijn het volledig integreerbare geval, dat behandeld werd door Frobenius, en het maximaal niet-integreerbare geval, dat behandeld werd door Darboux. Deze gevallen staan tegenwoordig bekend als respectievelijk de theorie der foliaties en de contactmeetkunde. In beide is er een eenvoudige lokale canonieke vorm voor het systeem, zonder enige invarianten. Een tot heden onovertroffen prestatie werd geleverd door E. Cartan, die in zijn uitgebreide studie *Les systèmes de Pfaff àcinq variables et les équations aux dérivées partielles du second ordre* in *Ann. Sci. Ecole Norm. Sup.* **27** (1910) voor het eerst een lokale invariant ontdekte, te weten een ternaire vorm van graad vier. En de symmetriegroep van een systeem waarvan de invariant gelijk aan 0 is, is de gespleten vorm van de uitzonderlijke