The textile industry may be characterised on the one hand by product-oriented thinking, and on the other by constructional skill in the lay out of the various equipment used in textile processing. In textile finishing there is generally a striking lack of knowledge of the principles of engineering. In the 1960s it was proposed by Meier-Windhorst in Germany [1] and by Parish in England [2] that chemical engineering principles be introduced into the mathematical description of textile finishing processes. The theory of transport phenomena, so successfully applied in chemical engineering, was advanced as a powerful means of aiding the understanding of the operation, design and innovation of textile finishing equipment. At the Twente University of Technology in Enschede a research programme was started on washing and (pre-)drying processes in which the scientific approach mentioned above was emphasised. The results were reported at earlier IFATCC congresses [3] and published in the literature [4–9].

This contribution is intended to give an idea of the methodology, as well as the results of the engineering
research carried out in collaboration with industry and TNO in our laboratory. The methodology used is detailed in this paper, and is summarised in Table 1.

One parameter is the specific transport phenomenon (momentum, heat and mass transfer) that is predominant in the process to be investigated. A second parameter is the engineering objective aimed at (operation, design and innovation). The combination of both parameters gives rise to different fields of interest of which three examples were selected.

The first example deals with the significance of fluid flow and pressure drop in the operation of a rotating washing machine. Secondly it is shown that knowledge of mass transfer in open-width washing machines leads to improved equipment design. In the third example the importance of transport phenomena in the innovation of equipment for pre-drying and drying of fabrics by means of porous rollers is demonstrated.

**OPERATION: MOMENTUM TRANSFER IN ROTATING WASHING MACHINES**

**Process Description**

The rotating washing machine consists of a perforated beam around which up to several thousand metres of fabric are wound (Figure 1). By rotating the batch and forcing wash liquor through the beam, impurities present in the fabric are removed.

![Figure 1 - Representation of a rotating washing machine](image)

Transport phenomena in a rotating batch of fabric exhibit a very complex character; the momentum transfer equation contains terms for pressure as well as centrifugal force and has to be solved in two (radial and axial) dimensions. Moreover mass transfer and momentum transfer are coupled by the velocity vector, and mass transfer has to be considered as a non-stationary process. Therefore a complete analytical solution of the relevant transfer equations is out of the question, and hence simplifying assumptions have to be made. One way of simplifying the process description is to consider momentum transport first in combination with statements of simple boundary conditions, such as constant volume flow through the fabric or constant initial pressure in the beam of the washing machine. It will be shown that momentum transfer, giving rise to hydrodynamic behaviour of equipment, is most important for the operation of the rotating washing machine. If in the batch of fabric axial flow is prevented (by centrifugal action and/or by sealing the sides of the batch) the momentum transfer equation reads as Eqn 1.

\[
\frac{dP}{dr} = -\frac{\mu}{K} v + \rho_0 w r
\]  

The continuity equation is given by Eqn 2.

\[
\frac{dv}{dr} + \frac{v}{r} = 0
\]  

Rearranging Eqn 1 and differentiating gives Eqn 3.

\[
\frac{dv}{dr} = \left( -\frac{dP}{dr} + \rho_0 w r \right) \frac{K}{\mu}
\]  

Combining Eqns 2 and 3 gives Eqn 4.

\[
\frac{d^2P}{dr^2} + \frac{1}{r} \frac{dP}{dr} = 2 \rho_0 w^2
\]  

The introduction of dimensionless variables:

leads to Eqn 5.

\[
\pi = \frac{P}{\frac{1}{2} \rho_0 w^2 R_i^3} \text{ and } x = \frac{r}{R_i}
\]

\[
\frac{d^2\pi}{dx^2} + \frac{1}{x} \frac{d\pi}{dx} = 4
\]  

or

\[
\frac{d}{dx} \left( \frac{x}{dX} \frac{d\pi}{dX} \right) = 4X
\]  

Eqn 5b has the general solution shown in Eqn 6.

\[
\pi = X^2 + C_1 \ln X + C_2
\]  

in which the constants \(C_1\) and \(C_2\) correspond with the boundary conditions applied. Irrespective of the operational conditions, the outlet pressure of the wash liquor is atmospheric, so that when \(X = R_i / R\), \(\pi\) will be zero.

JSDC Volume 101 July/August 1985 213
The second boundary value depends on the inlet conditions defined in Eqns 7 and 8.

\[ X = 1 \quad \pi = \pi_0 \text{ (constant inlet pressure)} \quad (7) \]

\[ X = 1 \quad \frac{d\pi}{dX} = 2 - \frac{2\mu v_o}{K \rho_0^2 R_1^2} \text{ (constant inlet velocity)} \quad (8) \]

For constant inlet pressure, the pressure and velocity profiles are represented by Eqn 9 (a and b).

\[ \pi = \pi_0 + X^2 - 1 - \frac{\left( \frac{R_z}{R_1} \right)^2 + \pi_0 - 1}{\ln(R_z/R_1)} \cdot \ln X \quad (9a) \]

\[ v = \frac{1}{\pi} \left( \frac{R_z}{R_1} \right)^2 + \pi_0 - 1 \cdot \frac{\rho_0^2 R_1}{\mu v_o} \cdot \frac{K}{X} \quad (9b) \]

For constant inlet velocity, the profiles follow from Eqn 10 (a and b).

\[ \pi = \frac{\mu v_o}{\frac{1}{2} \rho_0^2 R_1 K} \cdot \ln \left( \frac{R_z R_1}{X} \right) - \left( \frac{R_z}{R_1} \right)^2 - X^2 \quad (10a) \]

\[ v = \frac{1}{\pi} \quad (10b) \]

**Operational Characteristics**

It is clear that the relation between pressure drop \( \Delta P \) and flow rate \( \phi_0 \) of a rotating washing machine is independent of the boundary conditions applied. From Eqn 9 (a and b) and Eqn 10 (a and b), Eqn 11 may be derived.

\[ \pi_o = \frac{\mu \phi_0}{\frac{1}{2} \rho_0^2 R_1^2 K} \cdot \ln \left( \frac{R_z}{R_1} \right) - \left( \frac{R_z}{R_1} \right)^2 - 1 \quad (11) \]

which in dimensional form may be written in the form of Eqn 12.

\[ \Delta P = \frac{-\mu \phi_0}{2\pi L K a} \cdot \ln \left( \frac{R_z}{R_1} \right) \left( \frac{R_z}{R_1} \right)^2 - 1 \cdot \frac{1}{2} \rho_0^2 R_1^2 \quad (12) \]

From Eqn 12 it can be seen that the relationship between \( \Delta P \) and \( \phi_0 \) is represented by a straight line with a slope \( m \).

\[ m = \frac{\mu \ln(R_z/R_1)}{2 \pi K a} \]

and an intercept \( n \).

\[ n = -\left( \frac{R_z}{R_1} \right)^2 - 1 \cdot \frac{1}{2} \rho_0^2 R_1^2 \quad (Figure \ 2) \]

A washing machine and pump connected in series, in which the flow may be adjusted by a control valve, is illustrated in Figure 3. The hydrodynamic behaviour of the components of the system (pump, valve and washing machine) may be represented by individual head flow curves as shown in Figure 4. Arranging components in series is 'pressure additive', so that the system operation is indicated by point S on the pump characteristic (Figure 4) such that various pressure drops are related by Eqn 13.

\[ \Delta P \text{ (washing machine)} + \Delta P \text{ (valve)} = \Delta P \text{ (pump)} \quad (13) \]
The volume flow corresponding to the position of S is readily read-off on the abscissa in Figure 4.

When pump, valve and washing unit are arranged in parallel (Figure 5) a flow \( \phi_v \) recirculates through the valve whilst a flow \( \phi_0 \) is forced through the washing machine. If the suction pressure is atmospheric, the pressure drops across the components of the system are identical (Eqn 14).

\[
\Delta P \text{ (pump)} = \Delta P \text{ (valve)} = \Delta P \text{ (washing machine)} \tag{14}
\]

**Figure 6 – Head flow characteristics of Figure 5**

In Figure 6 the operating point S of the system is situated on the pump head curve, and flow rate is represented by Eqn 15.

\[
\phi_v \text{ (valve)} + \phi_0 \text{ (washing machine)} = (\phi_v + \phi_0) \text{ (pump)} \tag{15}
\]

**Industrial Application**

At constant inlet pressure, \( \pi \) can be written as a function of \( X \) as shown in Eqn 9a. The form of \( \pi = \pi(X) \) depends on the values of the parameters \( \pi_0 \) and \( R_v/R_i \); selecting the arbitrary values \( \pi_0 = 5 \) and \( R_v/R_i = 3 \), Eqn 9a is represented by the curve shown in Figure 7.

**Figure 7 – Dimensionless pressure \( \pi \) as a function of location \( X \)**

At a given value of \( X \) the pressure \( \pi \) becomes negative, causing penetration of air by suction from the environment; this results in unstable flow and staining of the fabric by air oxidation.

The condition under which negative pressures do not develop may be defined by requiring the location of the minimum pressure to be just at the outlet of the batch of fabric, so that:

\[
\frac{d\pi}{dX} = 0 \text{ and } X = R_v/R_i
\]

Differentiating Eqn 9a gives Eqn 16.

\[
\frac{d\pi}{dX} = 2X - \left( \frac{R_v}{R_i} \right)^2 \ln (R_v/R_i) \cdot \frac{1}{X} = 0
\]

From which

\[
X^2 = \left( \frac{(R_v/R_i)^2 + \pi_0^{-1} - 1}{2 \ln (R_v/R_i)} \right)^2 \tag{17}
\]

thus Eqn 17 is obtained.

\[
\pi_0 = 1 + \left( \frac{R_v}{R_i} \right)^2 \cdot \ln \left( \frac{R_v}{R_i} \right) - 1
\]

In industrial applications the following values of \( R_v \) and \( R_i \) are thought to be typical: \( R_i = 0.10 \, \text{m}, \, R_v/R_i = 3 \), so that \( \pi_0 = 11.8 \).

Now

\[
\pi_0 = \frac{P}{\frac{1}{\mu} \rho \omega R_i^2}
\]

from which follows that

\[
P_o = \pi_0 \cdot \frac{1}{\mu} \rho \omega R_i^2
\]

For example, with a rotation speed of 260 rev./min or \( \omega = 27.2 \, \text{rad/s}, \) and \( \rho = 965 \, \text{kg/m}^3 \) (water at 90°C), then the gauge inlet pressure is:

\[
P_o = 11.8 \times 10^4 \times 965 \times (27.2)^2 \times 0.10 = 4.2 \times 10^6 \, \text{N/m}^2
\]

From Eqn 11 the corresponding flow can be calculated as:

\[
1 + \left( \frac{R_v}{R_i} \right)^2 \ln \left( \frac{R_v}{R_i} \right) - 1 = \frac{\mu \omega \phi_o}{\frac{1}{\mu} \rho \omega R_i^2 \cdot 2\pi R_i^2 a} \cdot \ln \left( \frac{R_v}{R_i} \right) - \left( \frac{R_v}{R_i} \right)^2 - 1
\]

which may be simplified to Eqn 18.

\[
\phi_o = 2\pi R_v^2 a \frac{R_v}{R_i} \cdot \mu \omega \frac{K}{\mu}
\]

making

\[
L = 1.50 \, \text{m}
\]

\[
a = 0.01
\]

\[
K = 10^{-2} \, \text{m}^2
\]

\[
\mu = 0.3 \times 10^{-3} \, \text{kg/(m s)}
\]

and \( \phi_o = 2.0 \times 10^{-4} \, \text{m}^3/\text{s} \), a condition commonly in industrial practice.

**DESIGN: MASS TRANSFER IN AN OPEN-WIDTH WASHING MACHINE**

**Process Description**

Extraction of impurities from textiles is frequently carried out in an open-width washing machine, which consists of \( N \) washing units each provided with \( z \) compartments. Wash water and fabric flow countercurrently through the machine. Entrained liquid is removed by mangleing after each unit and is returned to the wash water flow (Figure 8).
For a mathematical description of the mass transfer in a compartment the following assumptions are made:

(a) Liquid flows (in wash water and in the fabric stream) leaving each compartment are in thermodynamic equilibrium

(b) Short-circuiting between successive compartments due to entrainment of liquid by the moving fabric is considered

(c) Liquid removed by mangle after each unit is returned to the final compartment of the corresponding unit.

The material balance for compartment \( z \) of unit \( N \) is described by Eqn 19.

\[
\begin{align*}
\phi_i \cdot c_{N,z-1} + (1-\lambda) \phi_i \cdot y_{N,z-1} + (1-\lambda) \phi_i y_{N,z} = \\
\text{Fabric stream} & \quad \text{Entrained liquor} & \quad \text{Wash water in} \\
\phi_i \cdot c_{N,z} + (1-\lambda) \phi_i \cdot y_{N,z} + [\phi_b + (1-\lambda) \phi_i] y_{N,z} = \\
\text{Fabric stream} & \quad \text{Entrained liquor} & \quad \text{Wash water out}
\end{align*}
\]

The equilibrium relation is represented by Eqn 20.

\[
y_{N,z} = k c_{N,z} \quad (20)
\]

Introduction of dimensionless variables

\[
R_{N,z} = \frac{c_{N,z}}{c_0} ; \quad \epsilon = \frac{\phi_b}{\phi_d} ; \quad a = 1 + (1-\lambda) \frac{\phi_i}{\phi_d} \cdot k
\]

where \( \epsilon \) = overall extraction factor

\( a \) = entrainment factor

results in Eqn 21.

\[
\frac{R_{N,z-2}}{R_{N,z}} = \frac{\epsilon + a}{a} = 1 + \frac{\epsilon}{a} \quad (21)
\]

For compartment \( z-1 \) of unit \( N \) the following balance is valid:

\[
\phi_i c_{N,z-1} + (1-\lambda) \phi_i y_{N,z-2} + [\phi_b + (1-\lambda) \phi_i] y_{N,z-1} = \\
\phi_i c_{N,z} + (1-\lambda) \phi_i y_{N,z-1} + [\phi_b + (1-\lambda) \phi_i] y_{N,z-2}
\]

or in dimensionless form as Eqn 22.

\[
a R_{N,z-1} + (\epsilon + a - 1) R_{N,z} = (\epsilon + 2a - 1) R_{N,z-1} \quad (22)
\]

From this:

\[
\frac{R_{N,z-2}}{R_{N,z}} = \frac{(\epsilon + 2a - 1)}{a} \cdot \frac{(\epsilon + a)}{a} - \frac{(\epsilon + a - 1)}{a}
\]

thus:

\[
\frac{R_{N,z-2}}{R_{N,z}} = 1 + \frac{\epsilon}{a} + \frac{\epsilon(\epsilon + 1)}{a} - \frac{\epsilon}{a - 1}
\]

giving Eqn 23

\[
\frac{R_{N,z-2}}{R_{N,z}} = 1 + \frac{\epsilon}{a} (1 + S) \quad (23)
\]

where \( S = \frac{\epsilon + a - 1}{a} \)

If the calculation up to the second compartment of unit \( N \) is continued then Eqn 24 is obtained.

\[
\frac{R_{N,1}}{R_{N,z}} = 1 + \frac{\epsilon}{a} (1 + S + S^2 + \ldots + S^{z-2}) = 1 + \frac{\epsilon S^{z-1}}{a - 1} \quad (24)
\]

For compartment 1 of unit \( N \) the material balance reads as:

\[
R_{N,0} + (\epsilon + a - 1) R_{N,1} = R_{N,1}(\epsilon + a)
\]

with

\[
\frac{R_{N,0}}{R_{N,z}} = 1 + \frac{\epsilon}{a} (1 + S + \ldots + S^{z-1}) = 1 + \frac{\epsilon}{a} \left( S^{z-1} - 1 \right) \quad (24)
\]

and

\[
\frac{R_{N,1}}{R_{N,z}} = 1 + \frac{\epsilon}{a} (1 + S + \ldots + S^{z-2}) = 1 + \frac{\epsilon}{a - 1} \left( S^{z-1} - 1 \right) \quad (24)
\]

from which Eqn 25 can be deduced.

\[
\frac{R_{N,0}}{R_{N,z}} = 1 + \frac{\epsilon}{a - 1} (\epsilon S^{z-1} - 1) \quad (25)
\]

Proceeding to compartment \( z \) of unit \( N-1 \) the material balance is shown in Eqn 26.

\[
\phi_i c_{N-1,z-1} + (1-\lambda) \phi_i y_{N-1,z-2} + [\phi_b + (1-\lambda) \phi_i] y_{N-1,z-1} = \\
\phi_i c_{N-1,z} + (1-\lambda) \phi_i y_{N-1,z-1} + [\phi_b + (1-\lambda) \phi_i] y_{N-1,z-2}
\]

or in dimensionless variables:

\[
a R_{N-1,z-1} = (\epsilon + a) R_{N-1,z} - \epsilon R_{N-1,1}
\]

Putting \( R_{N-1,1} = R_{N,1} \) and calculating \( R_{N,z} \) from Eqn 24, Eqn 27 can be obtained.

\[
\frac{R_{N-1,z-1}}{R_{N,z}} = 1 + \frac{\epsilon}{a - 1} (\epsilon S^{z-1} - 1) \quad (27)
\]

Similarly, the material balance for compartment \( z-1 \) of unit \( N-1 \) is:

\[
\frac{R_{N-1,z-2}}{R_{N,z}} = 1 + \frac{\epsilon}{a - 1} (\epsilon S^{z-1} - 1)
\]
Continuing the procedure just described, the expression for the first compartment of the first unit may be written as:

$$\frac{R_{e,0}}{R_{e,2}} = 1 + \frac{e}{e-1} \left[ e^{-1} S^{(e-1)} \right]$$

so that the non-extracted fraction is calculated as shown in Eqn 28.

$$I = \frac{e-1}{e^{Nz+1} S^{(e-1)} - 1} \quad (28)$$

The separation factor $S=\left( e + a - 1 \right)/a$ may be represented by a series of straight lines through the point $S=1$, $e=1$, as in Figure 9. This shows that for practical applications (Eqn 29):

$$S < e \quad \text{if} \quad e > 1 \quad (29)$$

**Design Strategy**

Eqn 28 is the starting point from which some design rules may be derived.

1. If the non-extracted fraction $I'$ is fixed by the requirement for a given product quality, four design variables are left: the extraction factor $e$, the separation including entrainment factor $S$, the number of units $N$ and the number of compartments per unit $x$.

2. The best extraction results will be obtained if the separation factor $S$ approaches the value of the extraction factor $e$, i.e. $S=e$. This may be realised by preventing liquid from becoming entrained by putting rollers on top of the upper guide rollers of the washing units; in that case $I'$ is defined by Eqn 30.

$$I' = \frac{e-1}{e^{Nz+1} - 1} \quad (30)$$

The number of design variables is thus reduced from four to two, namely the extraction factor $e$ and the total number of compartments $Nz$.

On the basis of Eqn 30, for a given quality $I'$, the total number of compartments required can be defined in the form of Eqn 31.

$$Nz = \frac{\log \frac{e-1 + I'}{I'}}{\log e} - 1 \quad (31)$$

For $e=1$ Eqn 30 simplifies to

$$I' = \frac{1}{Nz+1} \quad (32)$$

For a required product quality $I'=0.01$ (1% impurities left in the fabric) corresponds to nearly 100 compartments. The total number of compartments as calculated from Eqn 30 appears to be very sensitive to values of $e$ in the interval $1 < e < 2$, and rather insensitive to values of $e > 2$.

3. In practice some entrainment of liquid by the moving fabric is always present, so that Eqn 28 is valid. Inserting some values into this equation reveals that, for a given product quality $I'$ and a given entrainment factor $a$, the total number of compartments depends on the value of $N$; and increase of $N$ results in a slight decrease of $Nz$.

In view of the considerations given above, it is economically attractive to reduce the number of washing vessels, and increase the number of compartments per vessel. The investment costs of a washing machine are proportional to the number of washing vessels (including squeezer rollers), whilst the costs per vessel only slightly increase with the number of compartments per vessel. Therefore it may be good design to construct washing vessels containing a large number (e.g. 10–20) of compartments.

**Industrial Applications**

As already pointed out, investigation of Eqn 28 reveals the most favourable extraction performance to be obtained at a value of $e=2$. For design purposes on a commercial scale a few calculations should be carried out under the following conditions:

(a) Product quality required: $I'=0.01$

(b) Extraction factor: $e=k(\phi_v/\phi_H)=2$

(c) Entrainment factor: $a=1+(1-\lambda)k(\phi_v/\phi_H)=2$, which means with a reflux ratio $\lambda=0.5$ (at the upper roller of each compartment half of the liquid entrained is returned to the corresponding compartment) that, assuming the distribution coefficient $k$ of about one, the volume of flow of entrained liquid $\phi_v$ is about twice that of the liquid stream in the fabric $\phi_H$.

(d) For successive values of the number of washing vessels $N=1, 2, 3$, etc., the total number of compartments $Nz$ is calculated from Eqn 28.

The results of the calculations are given in Table 2.

<table>
<thead>
<tr>
<th>Extraction factor</th>
<th>Separation factor, $S$</th>
<th>Number of washing vessels, $N$</th>
<th>Number of compartments, $Nz$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.5</td>
<td>1</td>
<td>9.00</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>2</td>
<td>8.25</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>3</td>
<td>7.50</td>
</tr>
</tbody>
</table>

The differences in number of equilibrium compartments $Nz$ for different values of $N$ are marginal, hence the calculations confirm the previously stated need to maximise the number of compartments in one vessel.

In Table 2 the number of compartments calculated represent equilibrium stages, the conception of which may not apply to real compartments in industrial practice. Therefore it is very important to investigate the mixing conditions in compartments, which are to a large extent dependent on the ratio of residence time of the fabric to the mixing time.
(due to moving fabric and rotating roller) within a compartment. It may well be that the real compartments have an extraction efficiency (as compared with a theoretical stage) lower than 100%, and accordingly more compartments would have to be installed.

**INNOVATION: HEAT AND MASS TRANSFER IN POROUS ROLLERS [6]**

**Equipment Development**

In conventional textile finishing equipment the transfer between fabric and agent is normally carried out either co- or counter-currently in parallel flow.

The overall rate of transfer is greatly dependent on diffusion taking place within the fibres and in the yarns, as well as on transfer of heat and mass between liquid yarn and in the bulk liquid by convection. In Figure 10 these processes are illustrated.

![Figure 10 - Mass transfer steps in parallel flow](image1)

In parallel flow steps 1 and 2 may be intensified by convective means such as mixing and flow rate but the diffusional steps 3, 4 and 5 are not affected. Therefore new equipment has been devised in which cross flow is applied between fabric and process agent on a scale of distribution comparable with the size of the fibres. It may be expected that by convective action the rate of diffusion in voids, yarns and fibres will be greatly enhanced. The equipment proposed is a so-called porous roller consisting of bronze, steel, ceramics or plastics as a material of construction and produced by a specific sintering process (Figure 11).

![Figure 11 - Pre-drying and drying of fabric on a porous roller](image2)

**Tentative Process Descriptions**

As an alternative to mechanical removal of liquid in fabrics, e.g. by squeezers, porous rollers may be applied using air or preferably steam as the ‘blowing’ agent. During this pre-drying operation the following steps may be distinguished.

(a) Flow of liquid through the pores of the fabric, forced by the effect of the 'blowing' agent. The fluid velocity follows from the Darcy equation (Eqn 33).

\[ v = \frac{K_o \Delta p}{\mu x} \]  

(33)

from which the residence time required for one passage through the fabric may be calculated as shown in Eqn 34.

\[ t = \frac{\mu d^2}{2K_o \Delta p} \]  

(34)

(b) Removal of the liquid film at the interface between fabric and atmosphere. The available pressure drop \( \Delta p \) must meet the requirement shown in Eqn 35.

\[ \Delta p > \frac{2\sigma}{r} \]  

(35)

e.g. with \( \sigma = 0.07 \, N/m \) and \( r = 10^{-5} m \), then \( \Delta p > 0.14 \, bar \).

(c) Heating of the fabric by the blowing agent. The non-stationary heat transfer between steam and fabric may be described by the approximate expression given in Eqn 36.

\[ \frac{T_o - T_f}{T_o - T^*} = C_r \exp \left( -Fo \cdot Bi \right) \]  

(36)

in which \( Fo = at/d^4 \) (Fourier number) and \( Bi = ad/\lambda \) (Biot number). The time \( t \) required for heating the fabric up to the steam temperature is of the order of 50 ms!

(d) Evaporation of excess liquid in the pores by expansion of the blowing agent. At the heated fabric surface the expansion of steam causes a rapid evaporation of liquid present in the fabric. Consequently, after leaving the porous roller, depending on steam temperature, fabric velocity and air humidity, the liquid content of the fabric may be decreased by 5–15%.

Porous rollers prove to be very appropriate for the drying of textile materials and nonwovens. The mathematics of drying can be described by a combination of a material balance and a rate equation according to Eqns 37 and 38 respectively.

\[ \frac{\partial R}{\partial t} = \phi - \psi \frac{\partial y}{\partial z} + \epsilon_o \cdot \rho_i \frac{\partial y}{\partial t} \]  

(37)

Accumulation Convection Accumulation
in fabric in air in fabric

\[ \frac{\partial R}{\partial t} = K \rho_i \cdot a \left( y - y_f \right) f(R) \]  

(38)

Accumulation Transfer to air
in fabric

The accumulation term in air \( \epsilon_o \rho_i \frac{\partial y}{\partial t} \) is small compared with that in fabric \( \rho_i \left( 1 - \epsilon_o \right) \frac{\partial R}{\partial t} \), so that as a first approximation Eqn 39 can be written.

\[ \frac{\partial R}{\partial t} = \phi - \psi \frac{\partial y}{\partial z} = K \rho_i \cdot a \left( y - y_f \right) f(R) \]  

(39)

where \( f(R) = \frac{\text{actual drying rate}}{\text{rate of evaporation}} \)
In view of the experimental relation between drying rate \(-\frac{dR}{dt}\) and humidity \(R\) of the fabric, the drying region may be divided into a constant, transitional and falling rate periods respectively (Figure 12).

![Figure 12 - Different regions in the drying rate curve](image1)

The drying rate curve may be suitable for mathematical treatment by a linearising procedure as indicated in Figure 12. In that case the empirical function \(f(R)\) satisfies the following conditions:

\[
\begin{align*}
  f(R) &= 1 \quad \text{(constant rate period)} \\
  f(R) &= R \quad \text{(falling rate period)}
\end{align*}
\]

In both cases Eqn 39 may be solved analytically. It appears that depending on process conditions a substantial decrease in water content may be achieved within a time of exposure of the order of 1 s. Moreover the agreement between theoretical and experimental time of drying is very encouraging [6].

**Industrial Applications**

The porous roller seems to be very effective in processes in which a flowing agent has to be distributed on a small scale prior to intensive participation in transport phenomena and chemical reaction within the fabric. Therefore application of porous rollers seems to be appropriate in the following cases:

1. Rapid heating or cooling of a fabric may easily be carried out on one single porous roller (Figure 13(a))
2. Drying followed by thermofixation are processes readily adapted to a series arrangement of porous rollers (Figure 13(b))
3. Porous rollers may be suitable for the low-add-on techniques encountered in dyeing, coating and printing (Figure 13(c and d)).

**DISCUSSION AND CONCLUSIONS**

It has been shown that a gap existing between materials science and constructional skill is characteristic of the textile industry, and needs filling by process engineering principles. Transport phenomena (momentum, heat and mass transfer), which are commonly studied in chemical engineering problems, appear to be very promising for application to textile finishing processes.

The theory of transport phenomena not only seems to be appropriate to the proper understanding of operation and design of conventional equipment, but also is suitable for application to the development of new equipment. This sound theory combined with clear engineering objectives (operation, design, innovation) is a good starting point for engineering research.

Three combinations of transport phenomena with engineering objectives have been given and elaborated. The example of momentum transfer in the operation of a rotating washing machine showed the intimate relation between hydrodynamic behaviour and operational limits of a given piece of equipment. The second problem dealt with the coupling between mass transfer and design of an open-width washing machine. It was demonstrated that a thorough knowledge of mass transfer is of prime importance for improving the design and operation of conventional equipment. In the third example presented it was shown how a logical application of the theory of transport phenomena results in the innovation of new equipment (Mach nozzles and porous rollers), in which diffusion processes are accelerated by convectional flow on a scale comparable with the size of the fibres.

**REFERENCES**

LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Dimension</th>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>a</td>
<td>Hole fraction</td>
<td>m⁻¹</td>
<td>ρ</td>
<td>Density</td>
<td>kg m⁻³</td>
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<td>Bi</td>
<td>Biot number</td>
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<td>ρc</td>
<td>Fabric weight</td>
<td>kg m⁻²</td>
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<td>c,γ</td>
<td>Concentration</td>
<td>kg m⁻³</td>
<td>S</td>
<td>Separation factor</td>
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<td>I</td>
<td>Fraction non-extracted</td>
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<td>σ</td>
<td>Surface tension</td>
<td>N m⁻¹</td>
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<td>C</td>
<td>Concentration ratio</td>
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<td>T</td>
<td>Temperature</td>
<td>K, °C</td>
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<td>d</td>
<td>Fabric thickness</td>
<td>m</td>
<td>v</td>
<td>Velocity</td>
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<td>εo</td>
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<td>φ</td>
<td>Volume flow rate</td>
<td>m³ s⁻¹</td>
<td>X</td>
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<td>Fo</td>
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<td>y</td>
<td>Relative humidity of air</td>
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<td>k</td>
<td>Mass transfer coefficient</td>
<td>m s⁻¹</td>
<td>z</td>
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<td>K</td>
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<td>m⁻²</td>
<td>Indices</td>
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<td>L</td>
<td>Fabric width</td>
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<td>1, 2</td>
<td>Inner, outer position</td>
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<td>λ</td>
<td>Reflux from upper roller</td>
<td>m</td>
<td>b</td>
<td>Bath or wash liquid</td>
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<td>µ</td>
<td>Dynamic viscosity</td>
<td>kg m⁻¹ s⁻¹</td>
<td>d</td>
<td>Fabric</td>
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<td>N</td>
<td>Number of washing vessels</td>
<td>s⁻¹</td>
<td>f</td>
<td>Film entrained</td>
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<td>ω</td>
<td>Rotation speed</td>
<td>s⁻¹</td>
<td>g</td>
<td>Initial or overall</td>
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<td>ΔP, P</td>
<td>Gauge pressure</td>
<td>N m⁻²</td>
<td>h</td>
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<td>Air</td>
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COMMUNICATIONS

The Colour and Fastness of Natural Dyes of the Scottish Highlands

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From an extensive study of dyeings on wool using natural dye plants indigenous to the Highlands of Scotland, a selection representing some of the brighter colours obtained have been examined colorimetrically, and the fastnesses to washing and light recorded. The original colours and the colour changes during light fading have been recorded in Munsell and CIELAB (1976) colour coordinates. The fastnesses of these dyeings have been compared with the corresponding results previously reported for the more widely used natural dyes that formerly were traded internationally. The colours and colour changes are briefly discussed in terms of biosynthetic pathways, and related to the colours of yarns used in Scottish tartans of the past and present.

INTRODUCTION

The Highlands of Scotland have a long tradition of textile dyeing, but there are virtually no records of the source of dyes or the dyeing methods used in Scotland prior to 1750. However, the wearing of multicoloured clothing has been recorded in the western areas of Britain since Roman times, and the weaving of multicoloured checks or tartans has been practised in Scotland for at least 500 years [1]. It has been commonly assumed that locally grown plants provided the main source of colouring materials for the Highland dyer, although shipping records indicate that dyes were imported to the West of Scotland as early as the beginning of the 15th century [2].

Historians and textile conservators have an interest in the dyes and dyeing methods used in the past, and have used modern analytical techniques as an aid to identifying the dyes used in old textiles [3,4]. The present investigation was prompted by questions raised about important early collections of tartans held by the Museum of Tartans at Comrie in Perthshire. In particular, one commonly held view is that dyes extracted from native Scottish plants were only capable of producing plain or dull colours, and that for vividness of colour imported dyes had to be used. We have shown elsewhere [5,6] that dyes from plants native to Scotland can produce a range of quite bright colours. For a selection of these native dyes the present paper records the