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Note

Simple Perfect Square-Cylinders of Low Order

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The existence of two simple perfect square-cylinders of order 20 is discussed. So far, the lowest order simple perfect square-cylinders known, are of order 24, due to Wilson ("A Method for Finding Simple Perfect Squared Squarings," Thesis Univ. of Waterloo, 1967).

We report the existence of two simple perfect square-cylinders of order 20 shown in Figs. 1 and 2. Dissected cylinders are associated with graphs which we call cyl-nets. A cyl-net is obtained from a dissected cylinder in the following way. The two rims of the cylinder are associated with two vertices which are called "source" and "sink," respectively. Each horizontal line is associated with a vertex, while each square element is associated with an edge.

Since a dissected cylinder can be embedded on a sphere the associated cyl-net is planar. In case the cyl-net is 3-connected it must be a c-net. We are not interested in 1-connected graphs since they are associated with compound dissections; that is, two cylinders on top of each other. The class of 2-connected cyl-nets needs of course to be considered. This class can be constructed out of two cyl-nets of lower order. The order of a cyl-net is the number of square elements of the cylinder. Let the first cyl-net have a source SO1 and a sink SI1, and let $i1$ be a vertex adjacent with SI1, but different from SO1.

The second cyl-net has a source SO2, a sink SI2 and a vertex $i2$ adjacent to SI2, but different from SO2. Then a new cyl-net can be obtained by identifying $i1$ with $i2$ and SI1 with SI2 and removing either the edge ($i1$, SI1) or edge ($i2$, SI2). The new source is SO1, the new sink is SO2.

Another cyl-net can be obtained by removing both edges ($i1$, SI1) and ($i2$, SI2). In the same way one can identify $i1$ with SI2 and $i2$ with SI1, thus

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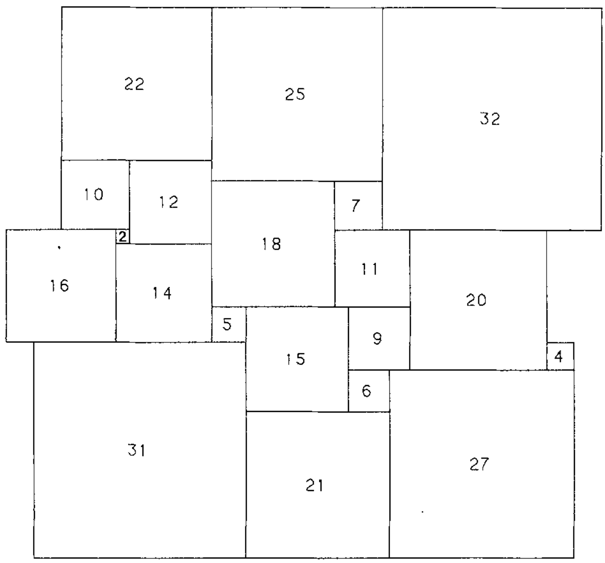

FIG. 1. Simple perfect square-cylinder, 79 × 79, order 20.

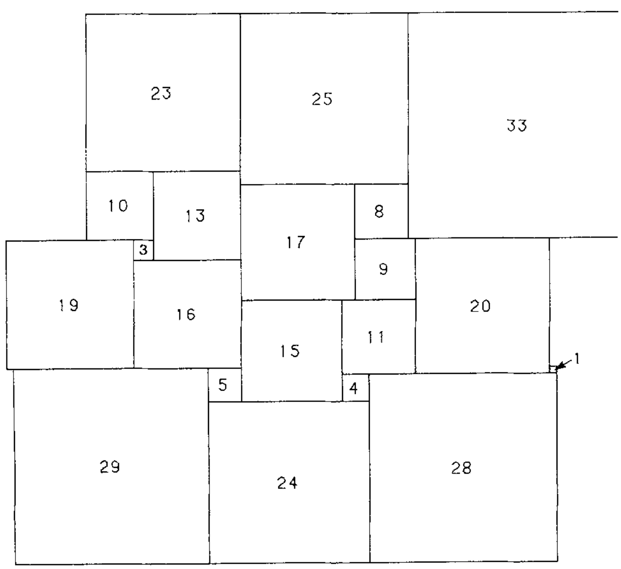

FIG. 2. Simple perfect square-cylinder, 81 × 81, order 20.

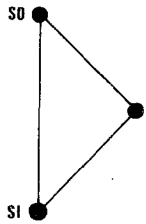

FIG. 3. The simplest 2-connected cyl-net.

obtaining another two new cyl-nets. The simplest 2-connected cyl-net is shown in Fig. 3.

Starting with the set Cn of all cyl-nets of order $\leqslant n$, we construct all cyl-nets of order $n + 1$ by combining two cyl-nets of Cn such that the resulting cyl-net is exactly of order $n + 1$. Furthermore cyl-nets originating from c-nets of order $n + 1$ are added to the set of cyl-nets of order $n + 1$. $Cn + 1$ clearly

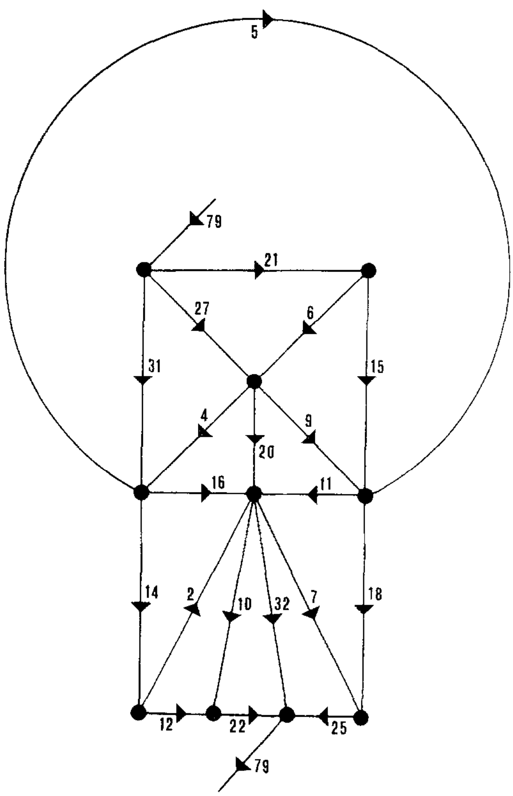

FIG. 4. C-Net with current sources corresponding with Fig. 1.

is the union of Cn and the set of cyl-nets of order $n + 1$. The set $C3$ contains just one element, namely, the simplest cyl-net of Fig. 3.

Dissected cylinders can be obtained from cyl-nets by considering them as electrical networks with unit resistors in their branches and applying a current source to the source and sink, respectively; the source and sink may not belong to a common face of the network otherwise the result will be a dissection of a rectangle (or square) into squares. The values of the currents in the branches are equal to the corresponding elements of the dissection.

Cyl-nets of orders up to and including 16 were generated and all possible dissectings were calculated by means of a computer. We did not find a simple perfect square-cylinder of order 16 or lower. Of orders up to and including 20 only those cyl-nets were investigated that originated from c-nets of corresponding orders. Among those we found two simple perfect square-cylinders both of order 20.

The two square-cylinders originate from the c-nets of Figs. 4 and 5. All c-nets of order 6 up to and including 22 are available on magnetic tape; they were generated by means of the methods described in [2, 3]. These c-nets were used to find the lowest order simple perfect squared square [4] and the

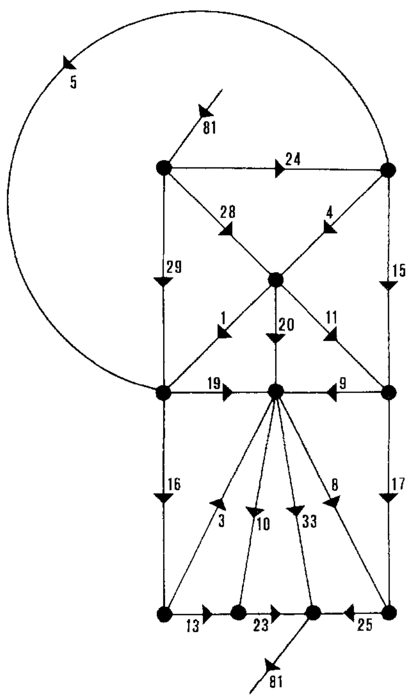

FIG. 5. C-Net with current sources corresponding with Fig. 2.

lowest order simple perfect 2×1 squared rectangle [5]. The c-nets were also used to prove that Willcock's 24 order compound squared square is of lowest order [6].

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