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Note

A Lowest Order Simple Perfect 2 × 1 Squared Rectangle

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The problem of dissecting a rectangle, whose horizontal side is twice the vertical side, into unequal squares (such a dissection is called *perfect*) in such a way that it does not contain a subset of squares arranged in a rectangle or square (such a dissection is called *simple*), has been treated by several authors [1, 2, 3].

P. J. Federico gave examples of several simple perfect 2×1 squared rectangles [3]. On September 2nd, 1978 a lowest-order simple perfect 2×1 squared rectangle was found. Its Bouwkamp code reads as follows:

(83, 49, 71, 69)(27, 22)(2, 67)(30, 65)(7, 20)(53, 24, 13)(11, 17, 5)(35)(29, 6)(23).

It is shown in Figure 1.

All 3-connected planar graphs (so-called *c*-nets) of orders 6 up to and including 22 are available on magnetic tape and magnetic disk. The order of a *c*-net is the number of its edges. These *c*-nets were used to find the lowest-order simple perfect squared square [4].

The *c*-nets were also used to search for the existence of solutions of 2×1 squared rectangles. It appears that there do not exist simple perfect 2×1 squared rectangles of orders lower than 22. The order of a squared rectangle is the number of its elements (squares). Below order 22 there only exist imperfect 2×1 squared rectangles. The number of simple imperfect 2×1 squared rectangle solutions is, however, small compared with the number of simple imperfect squared square solutions.

Since the least order simple 2×1 squared rectangle hitherto known was one of Federico's of order 23, I decided to generate *c*-nets of order 23 from the existing 22 order *c*-nets and test them for the existence of simple 2×1 squared rectangles.

There are 244 files each containing not more than 1000 *c*-nets of order 22. The number of 22 order *c*-nets is too large and the programs used (written in ALGOL–60) were too slow to process the complete set of 22 order *c*-nets.

Only 25 files were processed in this way. It is therefore not known whether there is another 22 order simple 2×1 perfect squared rectangle.

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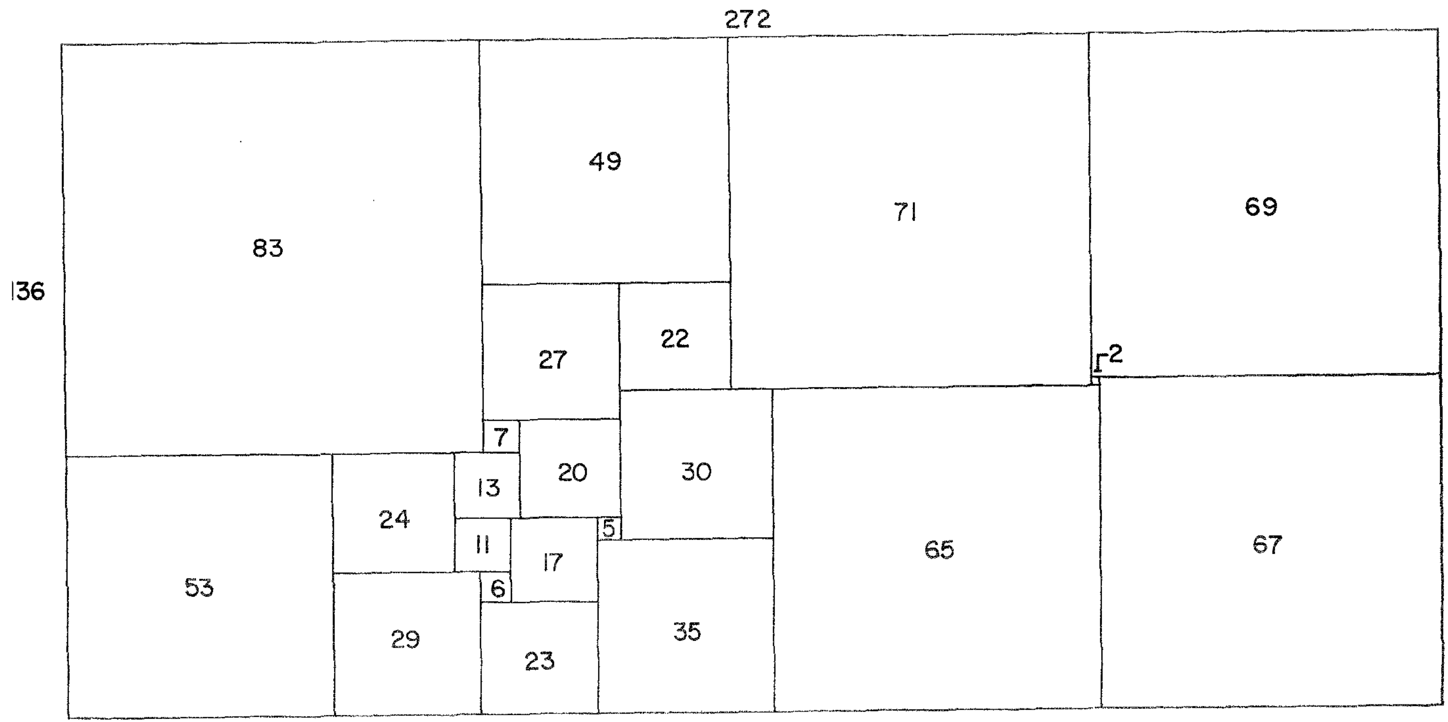

FIG. 1. A lowest-order simple perfect 2×1 squared rectangle.

I intend to investigate this possibility by modifying an existing Fortran program that exploits a much faster method [5]. The search also delivered two examples of simple perfect squared squares of order 22 [6].

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