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Performance and Convergence of Iterative Learning Control

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1 Abstract

Iterative Learning Control (ILC) [1] deals with the problem of finding the optimal input u^* to an unknown plant P by utilizing information from previous trials. The optimal input is defined in terms of the plant's output in the sense that it minimizes the distance between the actual output y and some desired output y_d. From a mathematical point of view, the problem boils down to defining a recursion relation on the space of inputs $\mathcal{U}$. This relation should define a convergent sequence and moreover it is generally required that $\lim_{k\to\infty} u(k) = u^*$.

At first sight, this seems like a difficult, if not impossible problem to solve. Nevertheless, many papers on ILC have addressed this problem and a variety of 'solutions' has been proposed [2]. The main idea that is common to all these solutions can easily be explained by means of a simple example.

Consider a recursion of the following type

$$u(k+1) \quad = \quad u(k) + e(k) \qquad (1)$$

where e is defined as $y_d - y$. Assume there exists a $u_d \in \mathcal{U}$ such that $Pu_d = y_d$. If this sequence is convergent, then necessarily $\lim_{k\to\infty} e(k) = 0$. Assuming that P is bounded on $\mathcal{U}$, it is not hard to show that a necessary and sufficient condition for convergence is given by

$$\|I - P\| < 1 \qquad (2)$$

This implies that P^{-1} exists and is bounded. In fact, if this condition holds, the recursion defined by (1) can be shown to converge to the fixed point $\bar{u} = P^{-1}y_d = P^{-1}(Pu_d) = u_d$. In the context of linear systems this means that, as a necessary condition for convergence, P should be invertible in $\mathcal{RH}^\infty$. Clearly only a very limited subclass of LTI plants satisfies this condition. This makes us wonder whether in general there exists at all a scheme that converges to the optimal input $u^* = u_d$. If the answer would turn out to be negative, the limits of performance have to be taken into account from the start, which means that aiming for perfect tracking is not a good idea.

In its full generality, there is no way we can answer this question. One way to constrain the problem is to consider only recursions that are linear in u.

In this presentation, we will propose a framework for the analysis of linear recursions of arbitrary order. Within this framework, the Iterative Learning Control problem reduces to a discrete time controller design problem on an infinite dimensional state space. Then, using the internal model principle, we are able to show that a zero steady state error can only be achieved if the controller has some integral action. This is illustrated in Figure 1 for the recursion defined by (1). We will elaborate on the implications of this result.

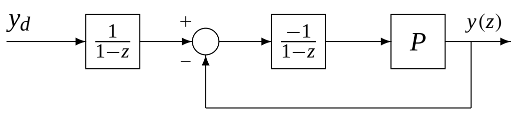

Figure 1: The internal model principle for ILC. The controller contains a model of the reference input.

References

[1] Kevin L. Moore, "Iterative Learning Control for Deterministic Systems," Springer-Verlag, 1993.

[2] Kevin L. Moore, J.-X. Xu (eds.), *International Journal of Control*, 73 (10), 2000. (Special issue on Iterative Learning Control)