UNIVERSITY OF TWENTE.

EXTENSIONS OF RESOURCE ALLOCATION PROBLEMS MOTIVATED BY SMART GRIDS

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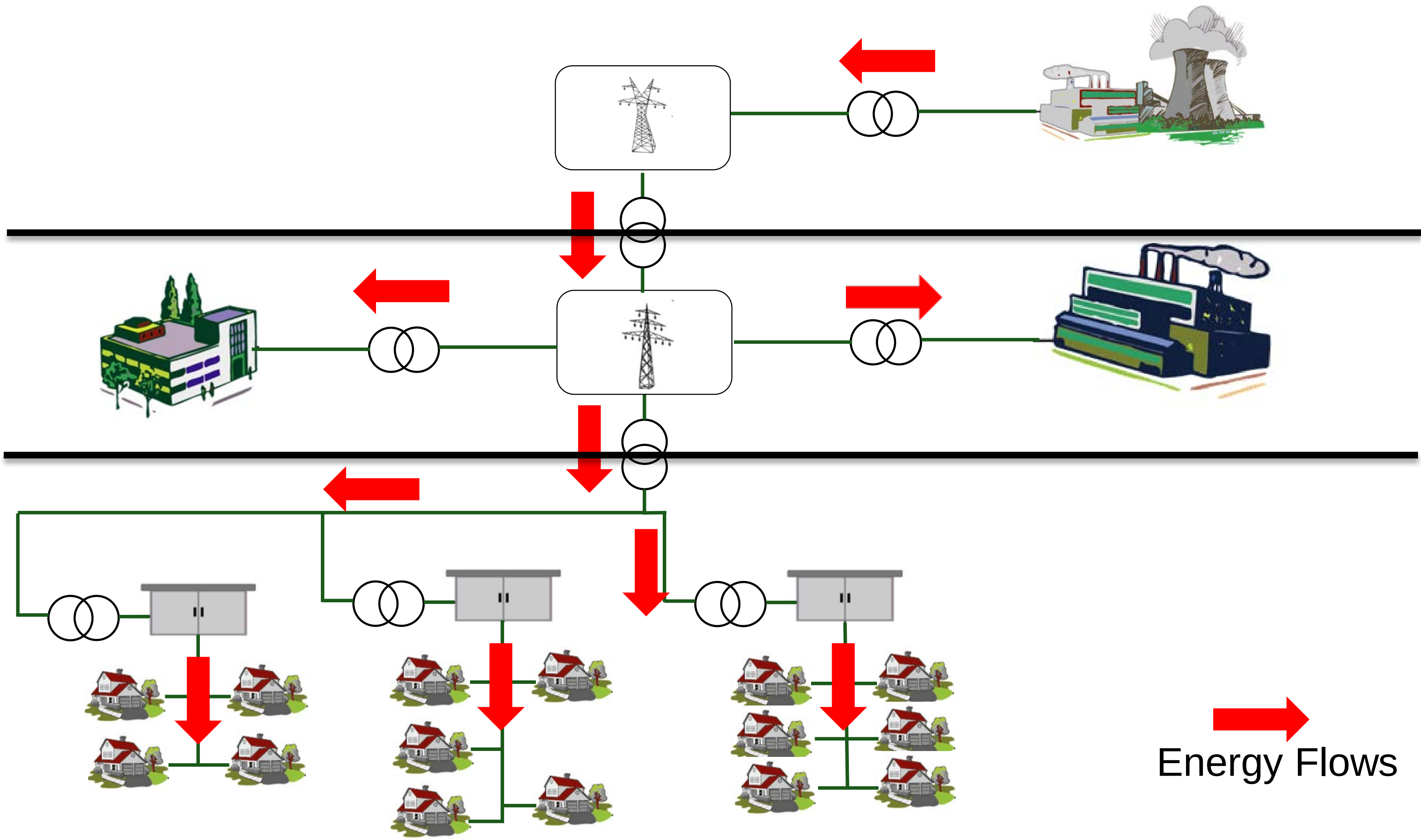
INTRODUCTION
POWER SYSTEMS OF THE 20TH CENTURY



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POWER SYSTEMS OF THE 21ST CENTURY – ENERGY TRANSITION

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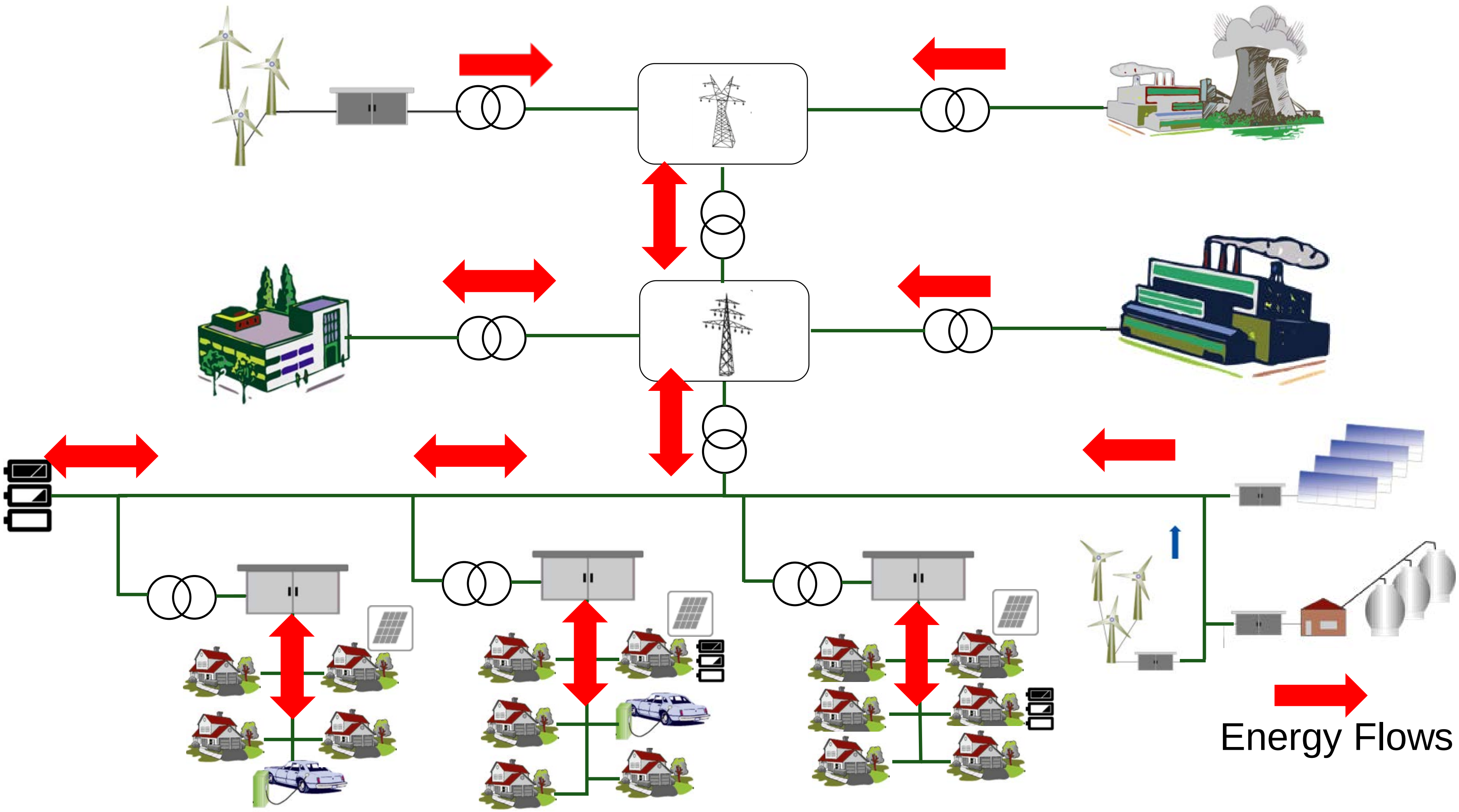
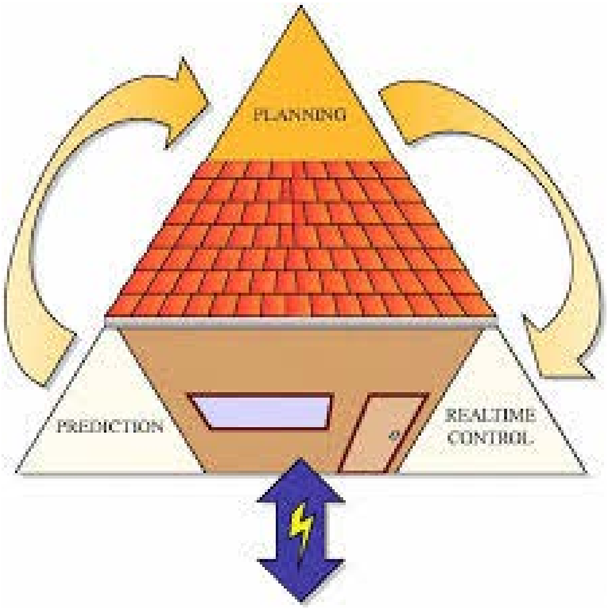


DECENTRALIZED ENERGY MANAGEMENT
TRIANA

Demand Side Management: Negotiate consumption patterns of controllable appliances through a (cooperative) coordination mechanism

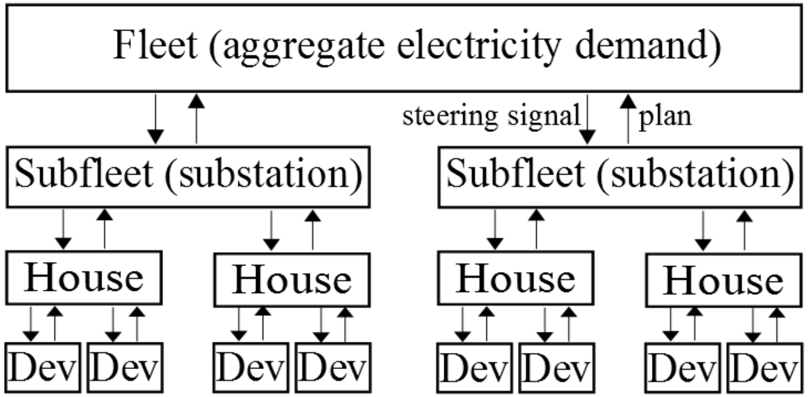
- Predict flexibility on house level
- Plan on neighbourhood level
- Account for difference with realtime control where needed

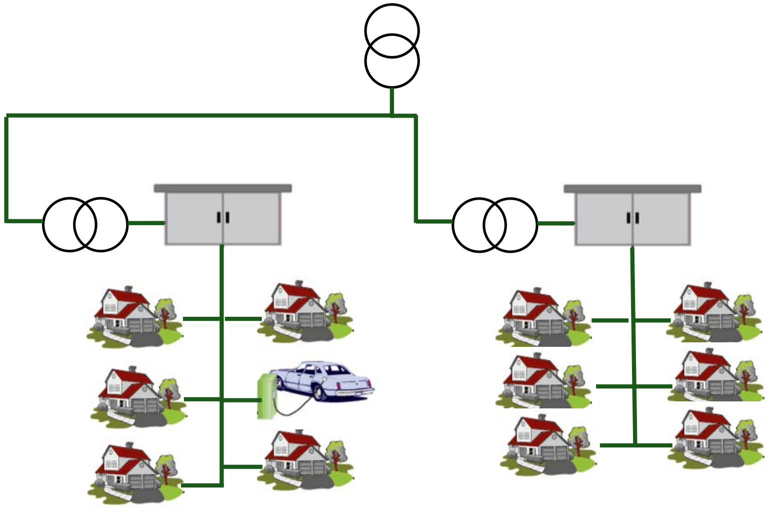


All done for fixed length time intervals, e.g., 15 minutes to align with power markets

DECENTRALIZED ENERGY MANAGEMENT
TRIANA – PROFILE STEERING

- Use structure of the grid





- Goal at the highest level may be e.g., peak shaving;
  - this gives a desired profile, e.g., flat profile

DEVICE LEVEL PLANNING
REMAINDER OF THIS TALK

- Consider two device level planning problems

- Electric Vehicle (EV)
  - Base case
  - Restricting the charging options

- Combined heat and power (CHP) with storage
  - Intermediate bounds on the storage

DEVICE LEVEL PLANNING
ELECTRIC VEHICLE – CHARGING PROBLEM

- Given:
  - N time intervals
  - EV needs to charge C units of energy
  - X_n^{max} max charge rate for interval n
  - $f_n(x_n)$ convex cost function for n
    - E.g.; $f_n(x_n) = (x_n - d_n)^2$, with d_n desired profile

- Problem:

$$\min_x f(x) = \sum_{n=1}^{N} f_n(x_n),$$

$$s.t. \sum_{n=1}^{N} x_n = C,$$

$$0 \leq x_n \leq X_n^{max} \quad \forall n.$$

DEVICE LEVEL PLANNING
ELECTRIC VEHICLE – CHARGING PROBLEM

- Problem is a form of resource allocation;
  - Separable convex objective
  - Convex constraint set
  - Resource constraint $\sum_{n=1}^{N} x_n = C_n$

- Well researched if constraints set is a bounding box
  - Optimality conditions (Gibbs' Lemma): There exists a λ with

$$f'_n(x_n) = \lambda \quad \Leftrightarrow \quad 0 < x_n < X_n^{max}$$

$$f'_n(x_n) \leq \lambda \quad \Leftrightarrow \quad x_n = X_n^{max}$$

$$f'_n(x_n) \geq \lambda \quad \Leftrightarrow \quad x_n = 0$$

DEVICE LEVEL PLANNING

ELECTRIC VEHICLE – OPTIMAL ALGORITHM

- Recall for profile steering; $f_n = (x_n - d_n)^2$
  - So $f'_n = 2(x_n - d_n)$

- Write x_n in terms of λ based on conditions:

$$x_n(\lambda) = \begin{cases} 0 & \text{if } \lambda \leq -2d_n \\ \lambda/_2 + d_n & \text{if } -2d_n < \lambda < 2(X_n^{max} - d_n) \\ X_n^{max} & \text{else} \end{cases}$$

- $x(\lambda) = \sum_{n=1}^{N} x_n$ piecewise linear increasing function of λ with breakpoints:

$$\{-2d_1, 2(X_1^{max} - d_1), \ldots, -2d_N, 2(X_N^{max} - d_N)\}$$

DEVICE LEVEL PLANNING

ELECTRIC VEHICLE – OPTIMAL ALGORITHM

$x(\lambda)$ with breakpoints $\{-2d_1, 2(X_1^{max} - d_1), \ldots, -2d_N, 2(X_N^{max} - d_N)\}$

- Find two adjacent breakpoints b_1 and b_2:

$$x(b_1) \leq C \leq x(b_2)$$

- Sort array and use binary search: $O(N \log N)$

DEVICE LEVEL PLANNING
ELECTRIC VEHICLE – LIMITING THE CHARGING OPTIONS

- EV cannot charge at all possible levels
  - Current situation, only few levels.

- Example: off,6A, 7A, … , 15A
  - This gives as feasible set for x_n

$$Z_n \coloneqq \{z_n^0, z_n^1, \dots, z_n^{m_n}\}$$

- So problem becomes:

$$\min_x f(x) = \sum_{n=1}^{N} f_n(x_n),$$
$$s.t. \sum_{n=1}^{N} x_n = C_n,$$
$$x_n \in Z_n \qquad \forall n.$$

DEVICE LEVEL PLANNING

ELECTRIC VEHICLE – DISCRETE EV CHARGING PROBLEM

$$\min_{x} f(x) = \sum_{n=1}^{N} f_n(x_n)$$

$$s.t. \sum_{n=1}^{N} x_n = C_n$$

$$x_n \in Z_n \qquad \forall n$$

Theorem

The discrete EV charging problem is NP-hard, even if all Z_n are equal

Proof based on even/odd partition

DEVICE LEVEL PLANNING
ELECTRIC VEHICLE – DISCRETE EV CHARGING PROBLEM

- Recall time intervals on minute scale
- Car switches much faster
  - Tests show car can react in ~4 sec
- Realistic case: allow convex combinations of charging levels
- Frequent switching of the charging level might harm the battery

DEVICE LEVEL PLANNING

DISCRETE EV – PIECEWISE LINEAR FORMULATION

$$\min_{y} \sum_{n=1}^{N} \sum_{j=0}^{m_n} y_n^j f_n(z_n^j)$$

$$s.t. \sum_{n=1}^{N} x_n = C$$

$$x_n = \sum_{j=1}^{m_n} z_n^j y_n^j \qquad \forall n$$

$$\sum_{j=1}^{m_n} y_n^j = 1 \qquad \forall n$$

$$y_n^j \geq 0 \qquad \forall n, j$$

DEVICE LEVEL PLANNING
DISCRETE EV – PIECEWISE LINEAR FORMULATION

- Problem is really just EV charging problem
  - With piecewise linear objective
  - Only use a piece if all other pieces with smaller slope are used

- Since f_n convex → slopes of pieces increase

- Leads to a greedy algorithm

DEVICE LEVEL PLANNING
DISCRETE EV – PIECEWISE LINEAR FORMULATION

- Let s_n^j be the slope of j-th piece:

$$s_n^j := \frac{f_n(z_n^{j+1}) - f_n(z_n^j)}{z_n^{j+1} - z_n^j}$$

- Step 1: Sort the array $S := \{s_1^0, s_2^0, \dots, s_N^0\}$.
- Step 2: - Maximally increase charging on interval of first slope
  - Delete this slope from S
  - Insert next slope of interval in S

- Repeat Step 2 while more charging needs to be done.

DEVICE LEVEL PLANNING

DISCRETE EV – PROPERTY OF THE SOLUTION

Lemma

There is an optimal solution to the piecewise linear approximation of the discrete EV problem that has $x_n \notin Z_n$ for at most one n.

- Follows directly from the optimal greedy algorithm

- The one ‘mistake’ can be approximated by a convex combination of the two allowed points around it

DEVICE LEVEL PLANNING
COMBINED HEAT AND POWER

- Combined heat and power unit (CHP)
  - Converts fuel → heat and electricity
  - Usually runs on gas!
  - produces heat demand of the house

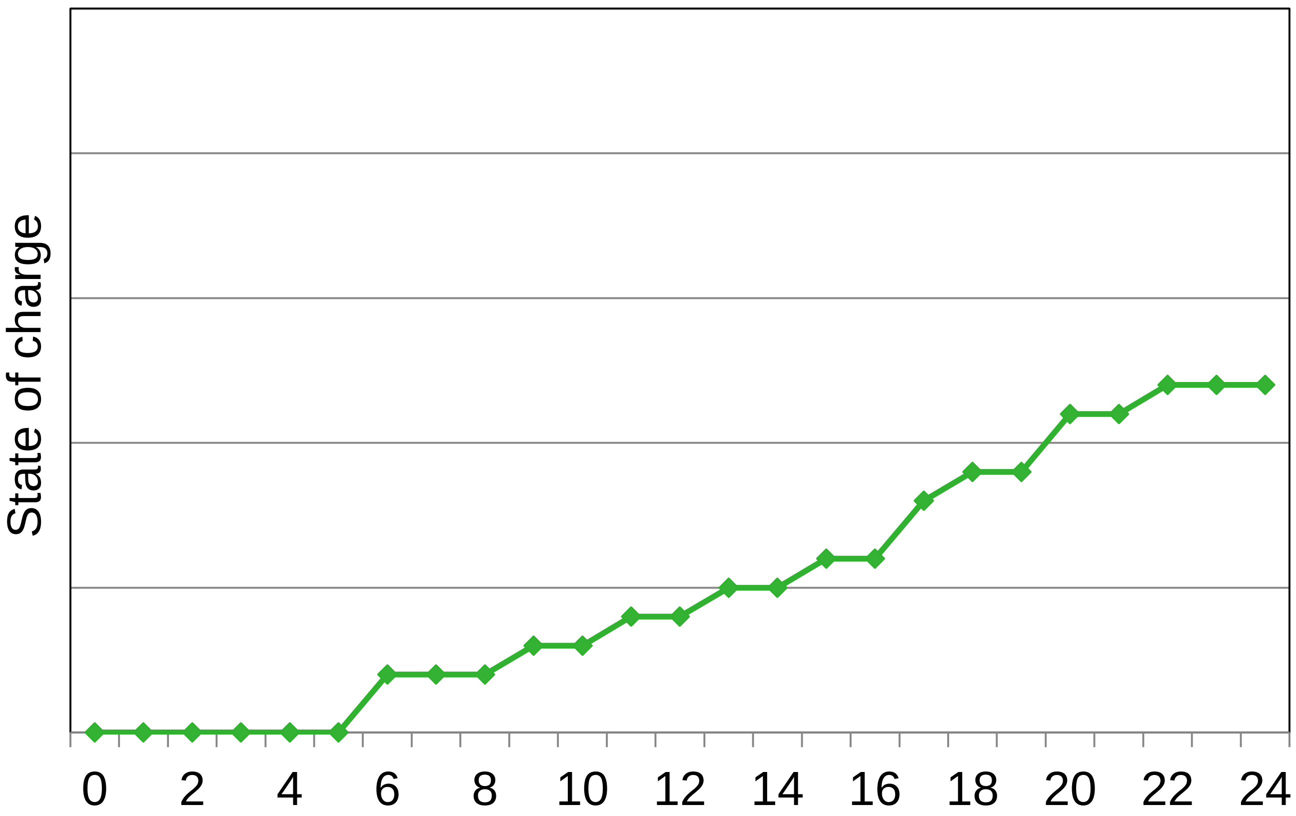
- Combined with heat storage a flexible device for electricity production

DEVICE LEVEL PLANNING
CHP – DEMAND

- Demand comes in events over the day
  - E.g., morning shower
  - Heating demand
  - →Leads to lower bounds on total production over time

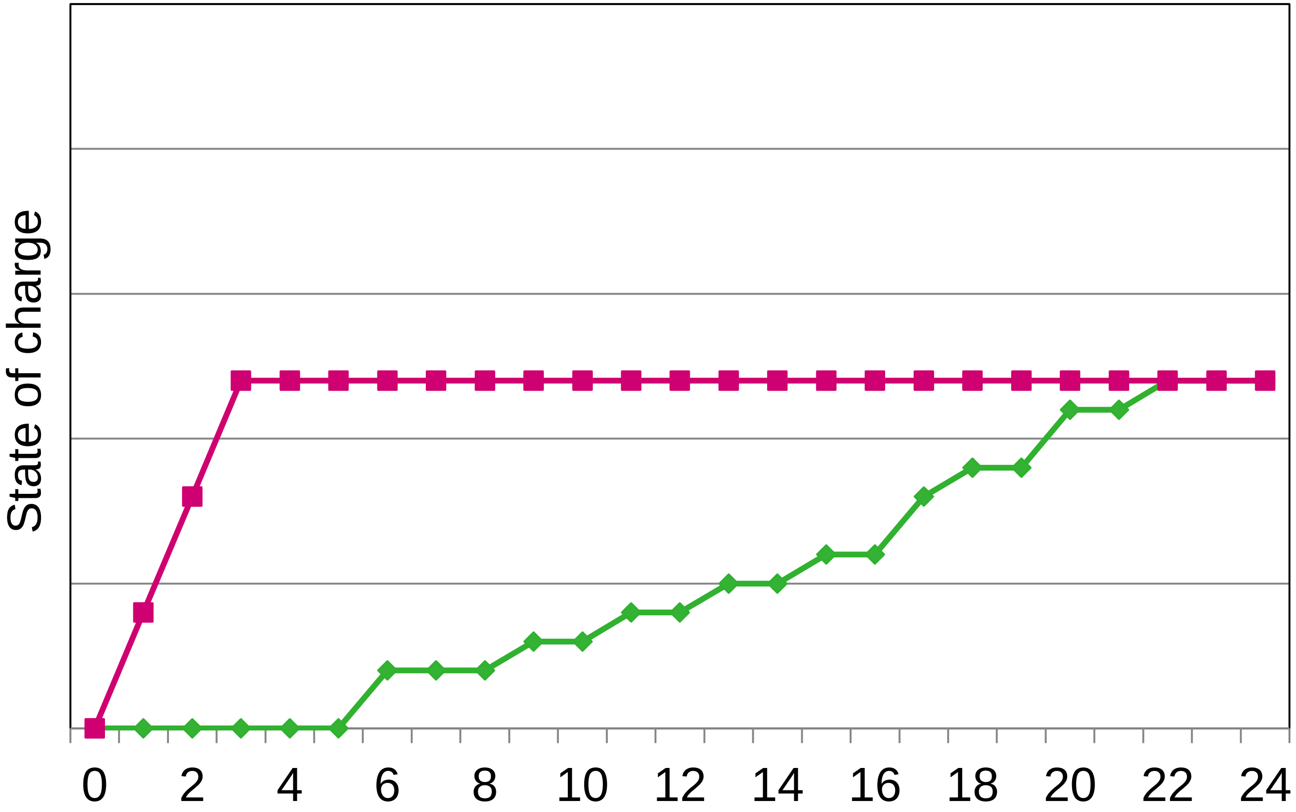
- Heat storage has limited capacity
  - → Leads to upper bounds on total production over time

DEVICE LEVEL PLANNING
CHP – STATE OF CHARGE BOUNDS



DEVICE LEVEL PLANNING

CHP – STATE OF CHARGE BOUNDS



DEVICE LEVEL PLANNING

CHP – PROBLEM FORMULATION

$$\min_x f(x) = \sum_{n=1}^{N} f_n(x_n),$$

$$s.t.\ B_n \leq \sum_{n'=1}^{n} x_{n'} \leq C_n \qquad \forall n,$$

$$0 \leq x_n \leq X_n^{max} \qquad \forall n.$$

- B_n and C_n are increasing sequences
- Can assume that $B_N = C_N$

DEVICE LEVEL PLANNING

CHP – OPTIMAL ALGORITHM

- Drop the cumulative bounds except for N
  - Then we have the EV problem again

- Let x be optimal for EV problem with k the interval with the worst violation
  - Surely the worst violation must be fixed

Lemma

Let y be optimal for the CHP problem then:

- If $\sum_{n=1}^{k} x_n > C_k \Rightarrow \sum_{n=1}^{k} y_n = C_k$
- If $\sum_{n=1}^{k} x_n < B_k \Rightarrow \sum_{n=1}^{k} y_n = B_k$

DEVICE LEVEL PLANNING

CHP – OPTIMAL ALGORITHM

- Allows for a recursive algorithm

- Step 1: Solve CHP problem without cumulative bounds
- Step 2: Split the problem on the largest violation of cumulative bounds

- Call algorithm on $1, \ldots, k$ and $k+1, \ldots, N$ separately

- In practise we expect few recursive calls!

CREDITS

ENERGY IN TWENTE: WWW.UTWENTE.NL/ENERGY