A Characterization of ET0L and EDT0L Languages*

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Abstract

There exists a PT0L language L_0 such that the following holds. A language L is an ET0L language if and only if there exists a mapping T induced by an a-NGSM (nondeterministic generalized sequential machine with accepting states) such that $L = T(L_0)$.

There exists an infinite collection of EPDT0L languages $D_{mn} \subseteq \Sigma_{mn}^\star$ ($n \geq m \geq 1$) such that the family EDT0L is characterized in the following way. A language L is an EDT0L language if and only if there exist $n \geq m \geq 1$, a homomorphism h and a regular language $R \subseteq \Sigma_{mn}^\star$ such that $L = h(D_{mn} \cap R)$.

1 Introduction

In studying sets closed under a fixed collection of operations —which are usually called algebras or algebraic structures— the sets generated (under those operations) by a finite number of elements have always obtained a considerable amount of attention. Thus in (semi)group theory much research has been done on finitely generated (semi)groups.

During the last few years families of languages closed under certain well-known operations have been intensively studied, which led to the introduction of the AFL concept (Abstract Family of Languages; cf. [7]). One of the major subjects in this field consists of the finitely generated or equivalently, the so-called principal (semi-)AFL's [8].

In this note we show that the family of ET0L languages can be obtained from a single PT0L language using only one derived operation instead of the complete collection of AFL-operations. As a direct consequence we obtain the well-known result that the family ET0L

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is a full principal AFL [3]. Moreover, we show a deterministic counterpart for the family EDT0L of Culik's [3] characterization of the family ET0L.

This note is organized in the following way. Section 2 contains some preliminaries from formal language theory and in particular from parallel rewriting. In section 3 we recall a characterization of the family ET0L due to Culik [3] which enables us to prove the existence of a PT0L language L_0 such that each ET0L language is the image of L_0 under an appropriate a-NGSM mapping (i.e., a mapping induced by a nondeterministic generalized sequential machine with accepting states). Section 4 deals with the deterministic analogue of Culik's result, characterizing the family EDT0L. Finally, in section 5 we will consider a few corollaries and applications.

2 Preliminaries

We refer to [12] or to [10] for terminology and basic facts from formal language theory and to [9] for elementary results in the theory of parallel rewriting.

A finite substitution τ over an alphabet V is a function mapping each symbol α from V into a nonempty finite language over V: $\tau(\alpha) \subseteq V^\star$. We extend τ in the usual way to words by $\tau(\lambda) = \{\lambda\}$ (λ stands for the empty word) and $\tau(\alpha_1 \cdots \alpha_n) = \tau(\alpha_1) \cdots \tau(\alpha_n)$ and to languages by $\tau(L) = \bigcup\{\tau(x) \mid x \in L\}$. A homomorphism over V is a particular kind of finite substitution, namely a substitution such that for each symbol α from V, the language $\tau(\alpha)$ contains exactly one word over V.

Definition. An *ET0L system* is a 4-tuple $G = (V, \Sigma, U, \omega)$ where V (called the alphabet of G) and $\Sigma \subseteq V$ (called the terminal alphabet) are finite sets. $\omega \in V^+$ is the initial word of G. U is a finite set of finite substitutions over V.

The language $L(G)$ generated by G is defined by $L(G) = (\bigcup \tau_{i_n} \cdots \tau_{i_1}(S)) \cap \Sigma^\star$, where the union is taken over all n-tuples $(i_1, \ldots, i_n)$ ($n = 0,\ 1,\ 2 \ldots$) with τ_{i_j} in U for each j ($1 \leq j \leq n$).

An *EDT0L system* is an ET0L system where all finite substitutions over V in U are homomorphisms over V. A *T0L* (*DT0L*) *system* is an ET0L (EDT0L) system with $V = \Sigma$. An X-system G is called *propagating* —denoted by PX— when all finite substitutions (or homomorphisms) involved in G are λ-free. Here X stands for ET0L, EDT0L, T0L and DT0L.

The family of languages generated by ET0L systems is denoted by ET0L. The same notational convention is applied to EDT0L, T0L and DT0L as well as to their propagating counterparts. □

Examples. (1) Let $G_{mn} = (V_{mn}, \Sigma_{mn}, U, S)$ where $\Sigma_{mn} = \{a_i \mid 1 \leq i \leq m\} \cup \{b_{kj}, b'_{kj} \mid 1 \leq k \leq n;\ j = 1,\ 2\}$; $V_{mn} = \Sigma_{mn} \cup \{S, F\}$ and $U = \{\tau_1, \tau_2\}$ for $n \geq m \geq 1$. The finite substitutions are defined for $j = 1,\ 2$ by

$$\begin{aligned}
\tau_j(S) &= \{b_{kj}Sb'_{kj}S \mid 1 \le k \le n\} \cup \{a_i \mid 1 \le i \le m\}, \\
\tau_j(a_i) &= \{F\} && \text{for each } i\ (1 \le i \le m), \\
\tau_j(\alpha) &= \{\alpha\} && \text{for } \alpha \in \{F\} \cup \{b_{kj}, b'_{kj} \mid 1 \le k \le n;\ j = 1, 2\}.
\end{aligned}$$

Clearly, $L(G_{mn})$ is an EPT0L language for each $n \ge m \ge 1$.

(2) Consider the PT0L system $G = (\Sigma, \Sigma, U, e)$ where

$\Sigma = \{c_1, c_2, d_1, d'_1, d_2, d'_2, e, e_1, e_2, f\}$

and $U = \{\tau_1, \tau_2, \tau_3\}$. The finite substitutions are defined by

$$\begin{aligned}
\tau_1(e) = \tau_1(e_1) = \tau_1(c_1) &= \{fd_1e_1d'_1c_1\}, \\
\tau_1(e_2) &= \{e_2, d_2e_2d'_2\}, \\
\tau_1(\alpha) &= \{\alpha\} && \text{for } \alpha \in \{c_2, d_1, d'_1, d_2, d'_2, f\}, \\
\tau_2(e) = \tau_2(e_2) = \tau_2(c_2) &= \{fd_2e_2d'_2c_2\}, \\
\tau_2(e_1) &= \{e_1, d_1e_1d'_1\}, \\
\tau_2(\alpha) &= \{\alpha\} && \text{for } \alpha \in \{c_1, d_1, d'_1, d_2, d'_2, f\}, \\
\tau_3(e_1) = \tau_3(c_1) = \tau_3(e_2) = \tau_3(c_2) &= \{e\}, \\
\tau_3(\alpha) &= \{\alpha\} && \text{for } \alpha \in \{d_1, d'_1, d_2, d'_2, e, f\}.
\end{aligned}$$

(3) Let $G^D_{mn} = (V_{mn}, \Sigma_{mn}, U, S_1)$ where Σ_{mn} is as in Example (1); $V_{mn} = \Sigma_{mn} \cup \{F, S_1, S_2, \ldots, S_n\}$ for $n \ge m \ge 1$.

Let Φ_n be the set of all total functions from $\{1, \ldots, n\}$ into $\{1, \ldots, n\} \times \{1, \ldots, n\}$; then clearly Φ_n is a finite set. U is defined by $U = \{\tau\} \cup \{\tau_{\varphi j} \mid \varphi \in \Phi_n;\ j = 1, 2\}$ whereas the finite substitutions on their turn are given for each φ in Φ_n and $j = 1, 2$ by

$$\begin{aligned}
\tau_{\varphi j}(S_k) &= \{b_{kj}S_pb'_{kj}S_q\} && \text{if and only if } \varphi(k) = (p, q)\ \ (1 \le k \le n), \\
\tau_{\varphi j}(a_i) &= \{F\} && \text{for each } i\ \ (1 \le i \le m), \\
\tau_{\varphi j}(\alpha) &= \{\alpha\} && \text{otherwise}, \\
\tau(S_i) &= \{a_i\} && \text{for each } i\ \ (1 \le i \le m), \\
\tau(\alpha) &= \{\alpha\} && \text{otherwise}.
\end{aligned}$$

Obviously $L(G^D_{mn})$ is an EPDT0L language for each $n \ge m \ge 1$. □

The following result on ET0L languages is due to Rozenberg [11] and has been generalized in [1] in order to cover for instance the EDT0L case.

Lemma 2.1. *For each ET0L (EDT0L) system with an arbitrary number of finite substitutions (homomorphisms) there exists an ET0L (EDT0L) system with only two finite substitutions (homomorphisms) generating the very same language.* □

Using a simple proof technique (cf. [3]) this can be easily modified into

Lemma 2.2. *Let $L \subseteq \Sigma^\star$ be an ET0L (EDT0L) language and let \$ be a symbol not in Σ. Then there exists an EPT0L (EPDT0L) system $G = (V, \Sigma \cup \{\$\}, U, S)$ with $U = \{\tau_1, \tau_2, \tau_3\}$ such that for each α in $V - \Sigma - \{\$\}$, $\tau_i(\alpha) \subseteq (V - \Sigma)^2$ $(i = 1,\ 2)$, $\tau_3(\alpha) \subseteq \Sigma \cup \{\$\}$, $\tau_i(\alpha) = \{\alpha\}$ if and only if $\alpha \in \Sigma \cup \{\$\}$ and $L = h_0(L(G))$, where h_0 is the homomorphism defined by $h_0(a) = a$ for a in Σ and $h_0(\$) = \lambda$.* □

A family of languages is called an *AFL* (Abstract Family of Languages) when it is closed under union, concatenation, Kleene $+$, λ-free homomorphism, inverse homomorphism and intersection with regular languages. An AFL is a *full AFL* when it is also closed under arbitrary homomorphisms. Each full AFL is closed under a-NGSM (and a-GSM) mappings —i.e., mappings induced by nondeterministic (deterministic, respectively) generalized sequential machines with accepting states— which we will define now.

Definition. A *nondeterministic generalized sequential machine with accepting states* (*a-NGSM*) is a 6-tuple $T = (Q, \Delta_1, \Delta_2, \delta, q_0, Q_F)$ where

- Q, Δ_1 and Δ_2 are finite sets (set of states, input alphabet, and output alphabet, respectively),
- $q_0 \in Q$ is the initial state,
- $Q_F \subseteq Q$ is the set of final states,
- δ is a mapping from $Q \times \Delta_1$ into the finite subsets of $Q \times \Delta_2^\star$.

We extend δ to a mapping from $Q \times \Delta_1^\star$ into finite subsets of $Q \times \Delta_2^\star$ as follows:

- $\delta(q, \lambda) = \{(q, \lambda)\}$,
- $\delta(q, \omega\alpha) = \{(q', \varphi_1\varphi_2) \mid \exists\, q'' :\ (q'', \varphi_1) \in \delta(q, \omega) \text{ and } (q', \varphi_2) \in \delta(q'', \alpha)\}$, where $q \in Q$, $\alpha \in \Delta_1$ and $\omega \in \Delta_1^\star$.

For each a-NGSM $T = (Q, \Delta_1, \Delta_2, \delta, q_0, Q_F)$ the function T from $\Delta_1^\star$ into subsets of $\Delta_2^\star$, defined by $T(\omega) = \{\varphi \mid (q, \varphi) \in \delta(q_0, \omega)$ for some $q \in Q_F\}$, is called an a-NGSM mapping. We extend the function T in the usual way to languages: $T(L) = \bigcup\{T(\omega) \mid \omega \in L\}$.

An a-NGSM is called *deterministic* (an *a-GSM*) when δ is a mapping from $Q \times \Delta_1$ into $Q \times \Delta_2^\star$. □

We conclude this section with a few elementary results concerning a-NGSM and a-GSM mappings. We start with a characterization of a-NGSM mappings; cf. [10].

Lemma 2.3. *A family K is closed under a-NGSM mappings if and only if K is closed under finite substitution and intersection with regular languages.* □

A similar characterization for a-GSM mappings does not hold; however, it is well known that, when a family is closed under a-GSM mappings, it is also closed under (arbitrary) homomorphism and intersection with regular languages.

Applying rather standard methods in machine theory, it is straightforward to prove (cf. [2])

Lemma 2.4. *The class of a-NGSM (a-GSM) mappings is closed under composition, i.e., if T_1 and T_2 are two a-NGSM's (a-GSM's), then there exists an a-NGSM (a-GSM, respectively) T, such that $T_2T_1(L) = T(L)$ for each language L.* □

Finally, we mention the properties of the families ET0L and EDT0L with respect to these machine mappings.

Lemma 2.5.

(1) *The family ET0L is closed under a-NGSM mappings.*

(2) *The family EDT0L is closed under a-GSM mappings, but not under a-NGSM mappings.*

Proof. (1) This is a direct consequence of the fact that ET0L is a full AFL [11].

(2) The closure of EDT0L under a-GSM mappings has been established in [5] and [1]. Since EDT0L is not closed under finite substitution [4], the second part of this proposition is immediately clear from Lemma 2.3. □

3 Characterization of ET0L Languages

Let $L_{mn} \subseteq \Sigma_{mn}^\star$ denote the language generated by the EPT0L system G_{mn} of Example (1). First we recall a result from [3] which enables us to establish the characterization we are looking for.

Theorem 3.1. [3] *A language L is an ET0L language if and only if there exist $n \geq m \geq 1$, a homomorphism h on $\Sigma_{mn}^\star$ and a regular language $R \subseteq \Sigma_{mn}^\star$ such that $L = h(L_{mn} \cap R)$.* □

We denote the language generated by the PT0L system G of Example (2) by L_0.

Theorem 3.2. *A language L is an ET0L language if and only if there exists an a-NGSM mapping T such that L equals the image of the PT0L language L_0 under T, i.e., $L = T(L_0)$.*

Proof. Since PT0L $\subseteq$ ET0L, we have by Lemma 2.5(1): $\{T(L_0) \mid T$ is an a-NGSM mapping$\} \subseteq$ ET0L.

In order to show the opposite containment, Theorem 3.1 implies that it suffices to prove that for each $n \geq m \geq 1$, L_{mn} can be obtained from L_0 by means of an appropriate a-NGSM mapping. Note that for each L in the family ET0L there exists an appropriate a-NGSM mapping T such that $L = T(L_{mn})$. Moreover, by Lemma 2.4, a-NGSM mappings are closed under composition.

Consider the a-NGSM $T_{mn} = (Q, \Sigma, \Sigma_{mn}, \delta, q_0, Q_F)$ where Σ and Σ_{mn} are the alphabets of L_0 and L_{mn}; cf. Examples (1) and (2), respectively. Let $Q_F = \{q_f\}$ and $Q = \{q_0, q_f\} \cup \{[k, j], \langle k, j\rangle \mid 1 \leq k \leq n;\ j = 1, 2\}$. The mapping δ is defined by

$$\begin{array}{ll}
\delta(q_0, e) = \{(q_f, a_i) \mid 1 \leq i \leq n\}, & \\
\delta(q_0, f) = \{(q_0, \lambda)\}, & \\
\delta(q_0, d_j) = \{([1, j], \lambda)\}, & \\
\delta([k, j], d_j) = \{([k{+}1, j], \lambda)\} & 1 \leq k \leq n{-}1, \\
\delta([k, j], f) = \{(q_0, b_{kj})\} & 1 \leq k \leq n, \\
\delta([k, j], \alpha) = \{(q_0, b_{kj} a_i) \mid 1 \leq i \leq m\} & 1 \leq k \leq n,\ \alpha \in \{c_1, c_2, e_1, e_2, e\}, \\
\delta(q_0, d'_j) = \{(\langle 1, j\rangle, \lambda)\}, & \\
\delta(\langle k, j\rangle, d'_j) = \{(\langle k{+}1, j\rangle, \lambda)\} & 1 \leq k \leq n{-}1, \\
\delta(\langle k, j\rangle, \alpha) = \{(q, b'_{kj} a_i) \mid q \in \{q_0, q_f\};\ 1 \leq i \leq m\} & 1 \leq k \leq n,\ \alpha \in \{c_1, c_2, e_1, e_2, e\}, \\
\delta(\langle k, j\rangle, f) = \{(q_0, b'_{kj} a_i) \mid 1 \leq i \leq m\} & 1 \leq k \leq n.
\end{array}$$

The effect of the a-NGSM mapping T_{mn} is rather simple. In the first place it changes any occurrence of c_1, c_2, e_1, e_2 or e nondeterministically into an a_i ($1 \leq i \leq m$). Secondly, it decodes strings like d_j^k and ${d'_j}^k$ into b_{kj} and b'_{kj} respectively whenever $1 \leq k \leq n$. But when T_{mn} reads in any particular input string a sequence of more than n consecutive occurrences of d_j (and d'_j) then it rejects that input word.

By a straightforward argument, which we leave to the reader, one can show that $L_{mn} = T_{mn}(L_0)$. □

Note that we coded each symbol b_{kj} (b'_{kj}, respectively) occurring in the language L_{mn} into $f d_j^k$ ($f {d'_j}^k$; cf. the definition of L_0), where the symbol f was introduced in order to separate the codes of different (occurrences of) b_{kj}'s (and b'_{kj}'s, respectively). When the numbers n and m are fixed an a-NGSM is able to perform the decoding process.

4 Characterization of EDT0L Languages

This section is devoted to establish a deterministic counterpart of Theorem 3.1.

For each $n \geq m \geq 1$, let D_{mn} denote the language generated by the EPDT0L system G^D_{mn} of Example (3).

Theorem 4.1. *A language L is an EDT0L language if and only if there exist $n \geq m \geq 1$, a homomorphism h on $\Sigma^\star_{mn}$ and a regular language $R_L \subseteq \Sigma^\star_{mn}$, such that $L = h(D_{mn} \cap R_L)$.*

Proof. Let $L \subseteq \Sigma^\star_{mn}$ be an EDT0L language and let $L = h_0(L(G))$ where $G = (V, \Sigma \cup \{\$\}, U, S)$ is the EDT0L system and h_0 is the homomorphism according to Lemma 2.2. Let $\Sigma_0 = \Sigma \cup \{\$\} = \{a_1, \ldots, a_m\}$ and let n be the number of nonterminal symbols in V, i.e., $V - \Sigma_0 = \{A_1, \ldots, A_n\}$. The alphabet Σ_{mn} was already introduced in Example (3). (Note that $\Sigma_0 \subseteq \Sigma_{mn}$.)

Consider the right-linear grammar $G_L = (V \cup \Sigma_{mn}, \Sigma_{mn}, P_L, S)$ where the set of productions is defined in the following way:

(1) if $\tau_3(A) = \{a\}$, then $A \to a$ is in P_L $(A \in V - \Sigma_0;\ a \in \Sigma_0)$,

(2) if $\tau_j(A_k) = \{BC\}$, $(j = 1,\ 2;\ A,\ B,\ C \in V - \Sigma_0)$, then

(2.1) $A_k \to b_{kj}B$ is in P_L, and

(2.2) $A \to ab'_{kj}C$ is in P_L for each A such that $\tau_3(A) = \{a\}$.

Now, let R_L be the regular language generated by G_L and let h_1 be the homomorphism on $\Sigma_{mn}^\star$ defined by

$$h_1(\alpha) = \alpha \quad \text{if } \alpha \in \Sigma_0,$$

$$h_1(\alpha) = \lambda \quad \text{if } \alpha \in \Sigma_{mn} - \Sigma_0.$$

By means of induction on the derivation length one can prove that $h_1(D_{mn} \cap R_L) = L(G)$. The proof is similar to the case of context-free languages (cf. [12] or [6]) or to the case of ET0L languages (cf. [3]) and is left to the reader.

We define the homomorphism h as the composition of h_0 (cf. Lemma 2.2) and h_1, i.e., $h(\alpha) = h_0h_1(\alpha)$ for α in Σ_{mn}. Clearly, $L = h(D_{mn} \cap R_L)$ holds for an arbitrary EDT0L language L.

Finally, since for each $n \geq m \geq 1$, $D_{mn} \in$ EPDT0L $\subseteq$ EDT0L and EDT0L is closed under arbitrary homomorphism and intersection with regular languages (Lemma 2.5 and the remark preceding Lemma 2.4), we have that $h(D_{mn} \cap R)$ is in EDT0L for each homomorphism h and each regular language R. □

5 Applications

Let SF denote the family of (context-free) sentential form languages, i.e., languages generated by grammars $G = (\Sigma, P, \omega)$ with $\omega \in \Sigma^+$, $P \subseteq \Sigma \times \Sigma^\star$, P is finite and the derivation relation "$\Rightarrow$" is as in the context-free case [13].

Example. (4) Let $G = (\Sigma, P, c)$ where $\Sigma = \{a, b, a', b', c\}$ and $P = \{c \to \lambda, c \to aca'c, c \to bcb'c\}$. Then $L(G)$ is an SF language and $L(G) \cap \{a, b, a', b'\}^\star$ is the Dyck language D_2 over $\{a, b, a', b'\}$. □

In a similar way we can define the family of regular sentential form languages RSF, i.e., languages generated by SF grammars under the restricted derivation relation "$\Rightarrow_R$", defined by $\varphi \Rightarrow_R \psi$ if and only if there exist α in Σ, ω_0, ω in $\Sigma^\star$ such that

(i) $\varphi = \omega_0\alpha$,

(ii) $\psi = \omega_0\omega$, and

(iii) (α, ω) is in P.

Example. (5) Consider $G = (\Sigma, P, a)$ where $\Sigma = \{a\}$ and $P = \{a \to aa\}$. Clearly, $L(G) = \{a^n \mid n \geq 1\}$. □

Now we recall a well-known result on context-free languages usually referred to as the Chomsky-Schützenberger Theorem; cf. [12] or [6].

Theorem 5.1. *For each context-free language L over an alphabet Σ of m symbols there exist an alphabet Σ_0 of $2m+4$ symbols, a Dyck language D_m over Σ_0, a homomorphism $h : \Sigma_0 \to \Sigma$ and a regular language $R \subseteq \Sigma_0^\star$ such that $L = h(D_m \cap R)$.* □

Let L_2 be the language generated by the SF grammar of Example (4). Completely analogous to the derivation of Theorem 3.2 from Theorem 3.1, we obtain from Theorem 5.1 the following characterization.

Theorem 5.2. *A language L is context-free if and only if there exists an a-NGSM mapping T such that L equals the image of the SF language L_2 under T, i.e., $L = T(L_2)$.* □

In [1] a family closed under finite substitution and intersection with regular languages, containing a nontrivial language (i.e., a language containing at least one nonempty word) was called a *prequasoid*. A prequasoid (full AFL) K is called *full principal* if and only if K contains a language L such that K equals the smallest prequasoid (full AFL, respectively)[1] that contains the language L. In that particular case the language L is called a *full generator* of K.

Corollary 5.3. *The families of ET0L, context-free, and regular languages are full principal prequasoids. Moreover, there exists a full generator in the corresponding family of sentential form languages (i.e., in T0L or even in PT0L, in SF, and in RSF, respectively).*

Proof. The former two cases are direct consequences of Theorems 3.2 and 5.2. The smallest prequasoid containing an arbitrary infinite regular language equals the family of regular languages [1]. In particular the RSF language of Example (5) is a full generator. □

This corollary immediately implies other well-known results, obtained by Culik [3] and Ginsburg & Greibach [8], respectively.

Corollary 5.4. *The families of ET0L, context-free and regular languages are full principal AFL's.* □

In order to get Theorem 3.2, starting from Theorem 3.1, we did the following:

(i) For each $n \geq m \geq 1$, we defined the homomorphism h_n by

[1] A full AFL K is full principal if and only if K is finitely generated, i.e., there exists a finite number of languages $L_1, L_2, \ldots, L_n$ such that K is the smallest full AFL that contains the languages $L_1, L_2, \ldots, L_n$. A similar result applies to full semi-AFL's and analogous types of languages families that are closed under union and a-NGSM mappings; cf. [8] or [7]. However, such an equivalence does not hold for full principal prequasoids because —although being closed under a-NGSM mappings— in general prequasoid are not closed under union.

$h_n(a_i) = e,$

$h_n(b_{kj}) = f d_j^k$ and

$h_n(b'_{kj}) = {d'_j}^k.$

(ii) It turned out that $\bigcup\{h_n(L_{mn}) \mid 1 \leq m \leq n\} = L_C$ is in the family ET0L.

(iii) It was even possible to replace L_C by a corresponding sentential form (i.e., T0L or even PT0L) language.

A similar procedure was also followed in case of context-free languages to obtain Theorem 5.2 from Theorem 5.1.

However, in the cases of EDT0L and E0L languages (i.e., languages generated by ET0L systems where consists of exactly one finite substitution), it seems to be very likely that such a construction is impossible, which may be caused by a non-containment of the language L_C in the original family.

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